# Electron assisted $dd$ reactions in metals

Péter Kálmán* and Tamás Keszthelyi
*Budapest University of Technology and Economics,*
*Institute of Physics, Budafoki út 8. F., H-1521 Budapest, Hungary*

The electron assisted low energy $dd$ reactions in deuterized metals are investigated. It is shown that if a metal is irradiated with slow, free deuterons then the $e+d+d \to e' + p + t$ and $e+d+d \to e' + n + {}^3He$ electron assisted $dd$ processes have measurable probabilities even in the case of slow deuterons. The cross sections and the yields in an irradiated sample are determined. The results are connected with the so called anomalous screening effect. It is concluded that the electron $dd$ processes have to be taken into account when evaluating the experimental data of low energy fusion reactions in metals.

PACS numbers: 25.45.-z, 25.60.Pj, 24.90.+d, 23.90.+w
Keywords: ${}^2H$-induced reactions, fusion reactions, other topics in nuclear reactions, other topics in radioactive decay and in-beam spectroscopy



Astrophysical factors have great importance in nuclear astrophysics [1], [2]. In the last two decades, investigating astrophysical factors of nuclear reactions of low atomic numbers, in the cross section measurements of the $dd$ reactions in deuterated metal targets extraordinary observations were made in low energy accelerator physics [3]. The phenomenon of increasing cross sections of the reactions measured in solids compared to the cross sections obtained in gaseous targets is the so called anomalous screening effect. A few years ago a systematical survey of the experimental methods applied in investigating and the theoretical efforts for the explanation of the anomalous screening effect was done [4] from which one can conclude that the full theoretical explanation of the effect is still open. Here it is shown that the electron assisted $dd$ processes in metal environment produce yields which are considerable in the energy range investigated in [4].

Let us consider the following $dd$ nuclear reactions

$$d + d \to \ p + t + \Delta, \tag{1}$$

$$d + d \to n + \ {}^3He + \Delta. \tag{2}$$

(Since the work [4] focuses to reactions (1) and (2) the $d + d \to {}^4He + \gamma$ reaction is not discussed in this paper.) Here $p$, $d$ and $t$ denote proton, deuteron and triton, respectively, and $\Delta$ is the energy of the reaction, i.e. the difference between the rest energies of the initial $(d + d)$ and final $\left( p + t \text{ and } n + {}^3He \right)$ states, $\Delta(pt) = 4.033\ MeV$ and $\Delta(nHe) = 3.269\ MeV$, respectively.

The energy dependence of the cross section ($\sigma$) of the processes (1) and (2) reads as

$$\sigma\left(E_r\right) = S\left(E_r\right) \exp\left[-2\pi\eta\left(E_r\right)\right] / E_r, \tag{3}$$

where $E_r$ is the relative kinetic energy of the deuterons, $S\left(E_r\right)$ is the astrophysical factor and $\eta\left(E_r\right)$ is the Sommerfeld parameter,

$$\eta\left(E_r\right) = \alpha_f \frac{m_0 c}{\hbar k\left(E_r\right)} = \alpha_f \sqrt{\frac{m_0 c^2}{2E_r}}, \tag{4}$$

where $k$ is the magnitude of the wave vector of the relative motion of the interacting deuterons in the center of mass coordinate system. The reduced rest mass $m_d/2$ of two deuterons of rest mass $m_d$ is approximated as $m_d/2 = m_0 c^2 = 931.494\ MeV$ which is the atomic mass (energy) unit. $\hbar$ is the reduced Planck constant, $c$ is the velocity of light and $\alpha_f$ is the fine structure constant [1], [2]. In the laboratory frame of reference the energy $E$ of the deuteron in the beam is $E = 2E_r$.

The $\exp\left[-2\pi\eta\left(E_r\right)\right]$ dependence of $\sigma\left(E_r\right)$ appears since the relative motion of the two deuterons is described by the Coulomb solution $\varphi(\mathbf{r}) \sim e^{-\pi\eta/2}\Gamma(1+i\eta)$, and the cross section of the process is proportional to

$$\left| e^{-\pi\eta/2}\Gamma(1+i\eta) \right|^2 = \frac{2\pi\eta\left(E_r\right)}{\exp\left[2\pi\eta\left(E_r\right)\right] - 1} = F_C(E_r). \tag{5}$$

Here $\Gamma$ is the Gamma function [5]. $F_C\left(E_r\right)$ is the square of the deuteron-deuteron Coulomb factor. The rate of nuclear reactions (1) and (2) decreases rapidly with decreasing beam energies in consequence of $F_C\left(E_r\right)$ becoming small.

However the situation changes in the case of the electron assisted versions of (1) and (2) which read

$$e + d + d \to e' + p + t + \Delta, \tag{6}$$

$$e + d + d \to e' + n + \ {}^3He + \Delta, \tag{7}$$

where $e$ denotes a conduction electron of the metal and $e'$ stands for it after assisting the reaction. The cross section of reactions (6) and (7) is proportional to the square of the electron-deuteron Coulomb factor expressed in the variable $E$

$$F_e\left(E\right) = \left| \frac{2\pi\eta_e\left(E\right)}{\exp\left[2\pi\eta_e\left(E\right)\right] - 1} \right| \tag{8}$$

*retired from Budapest University of Technology and Economics, Institute of Physics,
e-mail: kalmanpeter3@gmail.com

where

$$\eta_e(E) = -\alpha_f \frac{m_e c}{\hbar k_r(E)} = -\alpha_f \sqrt{\frac{m_0 c^2}{E}} \tag{9}$$

with $m_e$ the electronic rest mass. $k_r$ is the magnitude of the wave vector of motion in the center of mass coordinate system of the interacting deuteron and electron. Here the reduced mass $\mu = m_e m_d / (m_e + m_d)$ is approximated as $\mu = m_e$. In the cases ($5\ keV < E < 50\ keV$) investigated [4] $F_e(E)) = |2\pi\eta_e(E)|$. Although the processes (6) and (7) are second order processes it is expected that the disappearance of the small $\exp[-2\pi\eta(E)]$ quantity from the cross section of processes (6) and (7) may make the order of magnitude of their cross sections become comparable with or higher than the order of magnitude of (3) with decreasing $E$.

In the processes investigated a low energy deuteron beam bombs deuterized metal targets. The interaction between an ingoing deuteron and an electron of the metal is a Coulomb interaction of potential $V^{Cb}(\mathbf{x}) = \int -\left(2\pi^2\right)^{-1} e^2 \left(q^2 + q_s^2\right)^{-1} \exp(i\mathbf{q}\cdot\mathbf{x})\, d\mathbf{q}$ with coupling strength $e^2 = \alpha_f \hbar c$ and $q_s = \sqrt{4\pi e^2 \varrho(\varepsilon_F)}$ the Thomas-Fermi screening parameter, where $\varrho(\varepsilon_F)$ is the density of electron states at the Fermi-energy $\varepsilon_F$ in the metal [6]. The order of magnitude of $q_s$ is $10^8\ cm^{-1}$ for metallic elements. The other interaction that is taken into account between the nucleons of the deuterons is the strong interaction of potential $V^{St}(\mathbf{x}) = -f\exp(-s|\mathbf{x}|)/|\mathbf{x}|$, where the strong coupling strength $f = 0.08\hbar c$ [7] and $1/s$ is the range of the strong interaction.

In the second order process investigated a free electron of the conduction band of the metal takes part in a Coulomb scattering with an ingoing deuteron of the beam. In the intermediate state a virtual free neutron-proton pair is created in the (6) and (7) processes. The virtual neutron or proton corresponding to the processes (6) and (7), respectively, is captured due to the strong interaction by an other deuteron located in the solid. The reaction energy $\Delta$ is shared between the quasi-free final electron and the proton and tritium or neutron and $^3He$ corresponding to the processes (6) and (7), respectively. Since the aim of this paper is to show the fundamentals of the main effects, the simplest description is chosen.

The transition probability per unit time ($W_{fi}$) of the processes can be written as $W_{fi} = \frac{2\pi}{\hbar} \sum_f |T_{fi}|^2 \delta(E_f - \Delta)$ with $T_{fi} = \sum_\mu V_{f\mu}^{St} V_{\mu i}^{Cb} / \Delta E_{\mu i}$. Here $V_{\mu i}^{Cb}$ is the matrix element of the Coulomb potential between the initial and intermediate states and $V_{f\mu}^{St}$ is the matrix element of the potential of the strong interaction between intermediate and final states, furthermore $\Delta E_{\mu i} = E_\mu - E_i + \Delta_{\mu i}$. $E_i$, $E_\mu$ and $E_f$ are the kinetic energies in the initial, intermediate and final states, respectively. $\Delta_{\mu i}$ is the difference between the rest energies of the intermediate and initial states.

In reactions (6), (7) the Coulomb interaction virtually breaks up the deuteron and produces a virtually free proton and neutron. Correspondingly, $\Delta_{\mu i} = 2.225\ MeV$ which is the binding energy of the deuteron. The method of evaluation of matrix elements $V_{\mu i}^{Cb}$, $V_{f\mu}^{St}$ and the transition probability per unit time is similar to the method discussed recently [8] determining the cross section of electron assisted neutron exchange process.

The cross section $\sigma_{pt}$ of the electron assisted $d + d \to p + t$ process has considerable contributions if

$$|\mathbf{k}_1 + \mathbf{k}_2| \lesssim q_s \tag{10}$$

where $\mathbf{k}_1$, $\mathbf{k}_2$ are the wave vectors in the final state of the proton and the triton. In the cases which fulfill condition (10) $\mathbf{k}_1 = -\mathbf{k}_2 + \delta\mathbf{k}$ with $|\delta\mathbf{k}| = \delta k \sim q_s$ and $k_1 \simeq k_2 \simeq k_0 = \sqrt{(3/2)\, m_0 \Delta}/\hbar$, which is determined by the energy Dirac delta, resulting $\delta k / k_0 \sim q_s / k_0 \ll 1$. Evaluating $\sigma_{pt}$ the Weisskopf and the long wavelength approximations are used, i.e. for the initial and final bound neutron states we take $\Phi_W(\mathbf{r}) = \sqrt{3/(4\pi R^3)}$, if $|\mathbf{r}| \leq R$ and $\Phi_W(\mathbf{r}) = 0$ for $|\mathbf{r}| > R$, $R = \{R_d \text{ or } R_t\}$ where $R_d = 4.31\ fm$ is the radius of the deuteron and $R_t \simeq R_{^3He} = 2.26\ fm$ is the radius of the triton [9] and $\exp(-i\mathbf{k}_0 \cdot \mathbf{r}) = 1$ is used if $|\mathbf{r}| \leq R$. Furthermore $s = 1/r_0$ with $r_0 = 1.2 \times 10^{-13}$ $cm$ is applied. The further details of the calculation are similar to those made in [8]. So

$$\sigma_{pt} = C_{pt} \frac{u}{E} \tag{11}$$

with $E$ the kinetic energy of the deuterons in the beam, $u$ the deuteron over metal number densities and

$$C_{pt} = \frac{K_0}{\left[1 + \frac{2\Delta_{\mu i}}{3\Delta(pt)}\right]^2} \frac{N_c}{v_c q_s \Delta(pt)}, \tag{12}$$

where $v_c$ is the volume of the elementary cell of the metal, $N_c$ is the number of lattice sites in the elementary cell and $K_0 = 0.598 \frac{2^{12}}{3^6} \pi^2 \alpha_f^2 (0.08)^2 (R_d/R_t)^3 r_0^4 \left(m_0 c^2\right)^2$.

The cross section $\sigma_{nHe}$ of the electron assisted $d + d \to n + {}^3He$ reaction reads

$$\sigma_{nHe} = C_{nHe} \frac{u}{E} \tag{13}$$

where

$$C_{nHe} = \frac{K_0}{\left[1 + \frac{2\Delta_{\mu i}}{3\Delta(nHe)}\right]^2} \frac{N_c F_{pd}}{v_c q_s \Delta(nHe)} \tag{14}$$

with $F_{pd} = 2\pi\eta_{pd} / \left(e^{2\pi\eta_{pd}} - 1\right)$, which is the square of the $pd$ Coulomb factor before the action of strong interaction with $\eta_{pd} = \alpha_f \sqrt{2 m_0 c^2 / (3\Delta(nHe))}$.

The yield $(dN/dt)_{pt}$ of events of electron assisted $d + d \to p + t$ process in an elementary small volume $V_s$ of the sample can be written as $(dN/dt)_{pt} = N_t \sigma_{pt} \Phi$, where $\Phi$ is the flux of deuterons, $N_t$ is the number of target particles, i.e. the number of free electrons of the conduction band of the metal. The number of elementary cells in an elementary small volume $V_s$ of the sample is $V_s/v_c$ and the number of conduction electrons in an elementary cell is $g_e$ thus the number $N_t = g_e V_s / v_c$. Using

(11) the yield reads as $(dN/dt)_{pt} = g_e V_s C_{pt} u\Phi / (v_c E)$, where the quantities $\Delta(pt)$ and $E$ have to be substituted in $MeV$ units. The yield of the events of the electron assisted $d + d \rightarrow n + {}^3He$ reaction produced in an elementary small volume $V_s$ of the sample reads as $(dN/dt)_{nHe} = g_e V_s C_{nHe} u\Phi / (v_c E)$.

The yield $(dN/dt)_{usual}$ of the usual $d + d \rightarrow p + t$ process (without taking into account screening) can be written as $(dN/dt)_{usual} = N_c V_s \sigma(E) u\Phi / v_c$ where $N_c$ is the number of atoms in the elementary cell and $\sigma(E) = (2S(0)/E) \exp[-2\pi\eta(E)]$ with $\eta(E) = \alpha_f \sqrt{m_0 c^2 / E}$. Here $\sigma(E)$ is expressed in the variable $E = 2E_r$. In the extremely low energy range the $S(E_r) = S(0)$ approximation is valid and the $S(0)$ values of processes (1) and (2) are: $5.6 \times 10^{-2}$ and $5.5 \times 10^{-2}$ in $MeVb$ units [1].

It is useful to introduce the relative yield

$$r = \frac{\left(\frac{dN}{dt}\right)_{pt}}{\left(\frac{dN}{dt}\right)_{usual}} = \frac{g_e C_{pt}}{2N_c S(0)} \exp[2\pi\eta(E)], \qquad (15)$$

which is the ratio of the yields of electron assisted and normal $d + d \rightarrow p + t$ processes in an elementary volume of the sample. As model material we take $Pd$. It has $v_c = d^3/4$ with $d = 3,89 \times 10^{-8}$ $cm$ and $N_c = 2$. Using the data $\rho(\varepsilon_F) = 25$ $states/atom/Rydberg$ for the density of states at the Fermi energy of $Pd$ at $u = 0.3$ [4], [10], one obtains $q_s = 6.73 \times 10^8$ $cm^{-1}$ and $C_{pt} = 3.8 \times 10^{-13}$ $MeVb$. Furthermore, $g_e = 10$ and $r = 1.7 \times 10^{-11} \exp[2\pi\eta(E)]$ where $\eta(E) = \alpha_f \sqrt{m_0 c^2 / E}$ resulting $r = 1$ at $E = 3.186$ $keV$. From this number one can conclude that the yield produced by the electron assisted $d + d \rightarrow p + t$ process with decreasing beam energy becomes comparable with and larger than the yield produced by the normal $d + d \rightarrow p + t$ process near and below 3 $keV$. Since $C_{nHe} = 3.0 \times 10^{-13}$ $MeVb$ therefore $\sigma_{nHe}$ has the same order of magnitude as $\sigma_{pt}$ has and so similar statement can be made in the case of electron assisted $d + d \rightarrow n + {}^3He$ reaction too. Consequently, one can conclude that the electron assisted $d + d \rightarrow p + t$ and $d + d \rightarrow n + {}^3He$ processes should be taken into account when evaluating the data of low energy fusion reactions in metals.


[1] C. Angulo *et al.*, Nucl.Phys. A **656**, 3-183 (1999).

[2] P. Descouvemont, A. Adahchour, C. Angulo, A. Coc and E. Vangioni-Flam, At. Dat. Nucl. Dat. Tabl. **88**, 203 (2004).

[3] F. Raiola *et al.*, Eur. Phys. J. A **13**, 377-382 (2002); Phys. Lett. B **547**, 193-199 (2002); C. Bonomo *et al.*, Nucl. Phys. A**719**, 37c-42c (2003); J. Kasagi *et al.*, J. Phys. Soc. Japan, **71**, 2881-2885 (2002); K. Czerski *et al.*, Europhys. Lett. **54**, 449-455 (2001); Nucl. Instr. and Meth. B **193**, 183-187 (2002); A. Huke, K. Czerski and P. Heide, Nucl. Phys. A**719**, 279c-282c (2003).

[4] A. Huke *et al.*, Phys. Rev. C **78**, 015803 (2008).

[5] K. Alder *et al.*, Rev. Mod. Phys. **28**, 432-542 (1956).

[6] J. Sólyom, *Fundamentals of the Physics of Solids*, Vol. III., *Normal, Broken-Symmetry and Correlated Systems* (Springer, Berlin-Heidelberg, 2010).

[7] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[8] P. Kálmán and T. Keszthelyi, arXiv:1312.5498.

[9] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

[10] J. S. Faulkner, Phys. Rev. B **13**, 2391 (1976).