Cooperative internal conversion process

Péter Kálmán* and Tamás Keszthelyi†
Budapest University of Technology and Economics,
Institute of Physics, Budafoki út 8. F., H-1521 Budapest, Hungary

A new phenomenon, called cooperative internal conversion process, in which the coupling of bound-free electron and neutron transitions due to the dipole term of their Coulomb interaction permits cooperation of two nuclei leading to neutron exchange if it is allowed by energy conservation, is discussed theoretically. General expression of the cross section of the process is reported in one particle nuclear and spherical shell models as well in the case of free atoms (e.g. noble gases). A half-life characteristic of the process is also determined. The case of Ne is investigated numerically. The process may have significance in fields of nuclear waste disposal and nuclear energy production.

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I. INTRODUCTION

The issue of modifying nuclear processes by surroundings is a question of primary interest of nuclear physics from the very beginnings. In the last two decades, investigating astrophysical factors of nuclear reactions of low atomic numbers, in the cross section measurements of the *dd* reactions in deuterated metal targets extraordinary observations were made in low energy accelerator physics [1]. The phenomenon of increasing cross sections of the reactions measured in solids compared to the cross sections obtained in gaseous targets is the so called anomalous screening effect. A few years ago a systematical survey of the experimental methods applied in investigating and of the theoretical efforts for the explanation of the anomalous screening effect was made [2] from which one can conclude that the full theoretical explanation of the effect is still open. On the other hand, recently the effect of electron screening on the rate of α decay was theoretically investigated and observable effect was predicted [3].

However, the best known and investigated process in respect of modifying nuclear processes by surroundings is the internal conversion process, in which an atomic electron around an excited nucleus takes away nuclear transition energy of an electromagnetic multipole transition which otherwise would be prohibited by angular momentum conservation [4]. It was three decades ago that the direct modification of internal conversion process caused by intense radiation field was first studied [5]. In this field recently, among others, super intense few cycle x-ray pulse induced internal conversion process [6], resonance internal conversion as a way of accelerating nuclear processes [7] and excitation of the ^{229m}Th nuclear isomer via resonance conversion in ionized atoms [8] were theoretically investigated.

Motivated by the observations of the anomalous screening effect [1] we have searched for physical processes that may effect nuclear reactions in solid state environment. We theoretically found [9] that the leading channel of the $p+d \to {}^3He$ reaction in solid environment is the so called solid state internal conversion process, an adapted version of ordinary internal conversion process. It was shown [9] that if the reaction $p + d \to {}^3He$ takes place in solid material the nuclear energy is taken away by an electron of the environment instead of the emission of a γ photon.

In the usual internal conversion process only one nucleus is involved. However, very many pairs of nuclei can be found which may go to state of lower energy if they could cooperate exchanging e.g. a neutron [10]. Therefore it is worth investigating the way the electronic environment could lead to such cooperation.

Let us take two initial nuclei ${}^{A_1}_{Z_1}X$, ${}^{A_2}_{Z_2}Y$ and two final nuclei ${}^{A_1-1}_{Z_1}X$, ${}^{A_2+1}_{Z_2}Y$ which may be formed by neutron exchange from ${}^{A_1}_{Z_1}X$, ${}^{A_2}_{Z_2}Y$. If the sum E_{0i} of the rest energies of the initial nuclei is higher than the sum E_{0f} of the rest energies of the final nuclei, i.e. if $E_{0i} - E_{0f} = \Delta > 0$, then the two nuclei $\left({}^{A_1}_{Z_1}X \text{ and } {}^{A_2}_{Z_2}Y\right)$ are allowed to make a neutron exchange. $\Delta = \Delta_- + \Delta_+$, with $\Delta_- = \Delta_{A_1} - \Delta_{A_1-1}$ and $\Delta_+ = \Delta_{A_2} - \Delta_{A_2+1}$. Δ_{A_1}, Δ_{A_1-1}, Δ_{A_2}, Δ_{A_2+1} are the energy excesses of neutral atoms of mass numbers A_1, A_1-1, A_2, A_2+1, respectively [10]. As it was stated above there exist very many pairs of nuclei that fulfill the $\Delta > 0$ requirement. So it is a question of how these nuclei can realize the ${}^{A_1}_{Z_1}X$, ${}^{A_2}_{Z_2}Y \to {}^{A_1-1}_{Z_1}X$, ${}^{A_2+1}_{Z_2}Y$ neutron exchange transition.

The process

$$e + {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \to e' + {}^{A_1-1}_{Z_1}X + {}^{A_2+1}_{Z_2}Y + \Delta, \qquad (1)$$

that we are going to call cooperative internal conversion process (CICP), will be discussed with bound-free electron transition and for atomic state only. Numerically the case of Ne will be investigated. (In Eq.(1) e and e' denote bound and free electrons, respectively.)

*retired, e-mail: kalman@phy.bme.hu
†retired, e-mail: khelyi@phy.bme.hu

II. TRANSITION PROBABILITY PER UNIT TIME OF CICP IN SINGLE ELECTRON - SINGLE NUCLEON MODEL

The CICP can be understood with the aid of standard time independent second order perturbation calculation of quantum mechanics. There are two perturbations present which cause the effect and which have to be taken into account. The first is the electric dipole term V_{Cb}^{dip} of the Coulomb interaction between a bound electron and a neutron of an atom of nucleus ${}_{Z_1}^{A_1}X$ in which the effective charge of the neutron $q_n = -Z_1 e/A_1$ [11].

$$V_{Cb}^{dip} = \frac{Z_1 e^2}{A_1}\frac{4\pi}{3} x_1 x_e^{-2} \sum_{m=-1}^{m=1} Y_{1m}^*(\Omega_e) Y_{1m}(\Omega_1),$$

where Z_1 and A_1 are charge and mass numbers of the first nucleus, e is the elementary charge, x_1, x_e and Ω_1, Ω_e are magnitudes and solid angles of vectors $\mathbf{x}_1$, $\mathbf{x}_e$ which are the relative coordinates of the neutron and the electron in the first atom, respectively and Y_{1m} denotes spherical harmonics. The second is the nuclear strong interaction potential V_{st} of nucleus ${}_{Z_2}^{A_2}Y$. For this nuclear potential a rectangular potential well is assumed, i.e. $V_{st} = -V_0$ $(x_2 \leq R_{A_2})$ and $V_{st} = 0$ $(x_2 > R_{A_2})$ where x_2 is the magnitude of vector $\mathbf{x}_2$, which is the relative coordinate of the neutron in the second nucleus and R_{A_2} is its radius.

The initial state, which is composed from the state of a bound electron of the atom having nucleus ${}_{Z_1}^{A_1}X$, the state of a bound neutron of the atomic nucleus ${}_{Z_1}^{A_1}X$ and the states of centers of mass motion, is changed to first order due to the perturbation V_{Cb}^{dip} according to stationary perturbation calculation as

$$\begin{aligned}\psi_i \;=\; & [\psi_i^{(0)} + \sum_k a_{bb,k}\psi_{k,bb}^{(0)} \\ & + \sum_k \int a_{bf,k}\psi_{k,bf}^{(0)} d\nu_k + \int a_{ff,k}\psi_{k,ff}^{(0)} d\nu_k] e^{-\frac{iE_i t}{\hbar}}\end{aligned} \tag{2}$$

Here E_i is the total initial energy, which includes the sum E_{0i} of the rest energies of the initial nuclei ${}_{Z_1}^{A_1}X$, ${}_{Z_2}^{A_2}Y$. $\psi_i^{(0)}$ is the product of the unperturbed bound electron and neutron states and two plane waves ψ_{CM,A_1} and ψ_{CM,A_2} which describe the motions of the centers of mass of the atoms having ${}_{Z_1}^{A_1}X$ and ${}_{Z_2}^{A_2}Y$ nucleus, respectively. $\psi_{k,bb}^{(0)}$ is the product of other bound electron and neutron states, ψ_{CM,A_1} and ψ_{CM,A_2}. In $\psi_{k,bf}^{(0)}$ one of the electron and neutron states is bound and the other is free which are multiplied by ψ_{CM,A_1} (in case of free electron) or by ψ_{CM,A_1-1} (in case of free neutron) and ψ_{CM,A_2}, where ψ_{CM,A_1-1} describes the motion of the center of mass of the atom having ${}_{Z_1}^{A_1-1}X$ nucleus. The last term is interesting from the point of view of our process and in $\psi_{k,ff}^{(0)}$ both the electron and the neutron are free and their product is multiplied by ψ_{CM,A_1-1} and ψ_{CM,A_2}. Accordingly

$$\delta\psi_{i,free} = \int \frac{V_{Cb,ki}^{dip}}{E_i - E_k}\psi_{k,ff}^{(0)} d\nu_k e^{-\frac{iE_i t}{\hbar}}. \tag{3}$$

The sum of the energy of the free electron, neutron and center of mass states is E_k, which contains the sum E_{0k} of the rest energies and the state density is $d\nu_k$ [12].

Taking into account the interaction V_{st} between a free neutron and an other nucleus ${}_{Z_2}^{A_2}Y$ the second order transition probability per unit time reads as

$$W_{fi} = \frac{2\pi}{\hbar}\int \left| \int \frac{V_{st,fk} V_{Cb,ki}^{dip}}{E_i - E_k} d\nu_k \right|^2 \delta\left(E_i - E_f\right) d\nu_f. \tag{4}$$

Here $V_{Cb,ki}^{dip}$ and $V_{st,fk}$ are matrix elements of V_{Cb}^{dip} and V_{st} with states $\psi_{k,ff}^{(0)}$, $\psi_i^{(0)}$ and $\psi_f^{(0)}$, $\psi_{k,ff}^{(0)}$, respectively. $\psi_f^{(0)}$ is the product of the free electron state, the bound neutron and free center of mass state of nuclei ${}_{Z_1}^{A_1-1}X$ and ${}_{Z_2}^{A_2+1}Y$. The quantity $d\nu_f$ is the density of the final states of sum energy E_f, which comprises the sum E_{0f} of the rest energies of the final nuclei ${}_{Z_1}^{A_1-1}X$ and ${}_{Z_2}^{A_2+1}Y$. If the free states are plane waves of wave vectors $\mathbf{k}_e$, $\mathbf{k}_1$ and $\mathbf{k}_2$ corresponding to the wave vectors of the free electron, the ${}_{Z_1}^{A_1-1}X$ nucleus which has lost the neutron and the nucleus ${}_{Z_2}^{A_2+1}Y$ which has taken up the neutron then

$$d\nu_f = \frac{V^3}{(2\pi)^9} d\mathbf{k}_e d\mathbf{k}_1 d\mathbf{k}_2,\; d\nu_k = \frac{V}{(2\pi)^3} d\mathbf{k}_n,$$

where $\mathbf{k}_n$ is the wave vector of the free (intermediate) neutron, and V is the volume of normalization. The initial wave vectors of atoms having nuclei ${}_{Z_1}^{A_1}X$ and ${}_{Z_2}^{A_2}Y$ are negligible. (One can say that the initial bound neutron of nucleus ${}_{Z_1}^{A_1}X$ is excited into an intermediate free state due to the dipole term V_{Cb}^{dip} of its Coulomb-interaction with one of the bound atomic electrons and from this intermediate state it is captured by an other nucleus ${}_{Z_2}^{A_2}Y$ due to its nuclear potential V_{st} created by strong interaction forming the nucleus ${}_{Z_2}^{A_2+1}Y$ in this way.) The nuclear energy difference Δ, that is the reaction energy, is distributed between the kinetic energies of the final free electron and the two final nuclei. All told, in Eq.(1) the nucleus ${}_{Z_1}^{A_1}X$ looses a neutron which is taken up by the nucleus ${}_{Z_2}^{A_2}Y$ forming ${}_{Z_2}^{A_2+1}Y$ in this manner.

III. CROSS SECTION OF CICP IN SINGLE ELECTRON - SINGLE NUCLEON MODEL

The cross section $\sigma_{bf}(A_1, A_2)$ of the bound-free (bf) electron transitions of CICP can be determined from the transition probability per unit time by standard method. The evaluation of the transition probability per unit time is carried out in one particle nuclear model. Hydrogen

like state of binding energy E_{Bi} and Coulomb-factor corrected plane wave are used as initial, bound and final, free electron states. The motion of the intermediate neutron and the two final nuclei are also described by plane waves. The motions of the centers of mass of the two atoms are taken into account. The rest masses of the two initial nuclei of mass numbers A_1 and A_2 are $m_1 = A_1 m_0$ and $m_2 = A_2 m_0$ where $m_0 c^2 = 931.494\ MeV$ is the atomic energy unit. The cross section of the process is approximated with $\sigma_{bf}(A_1, A_2)$, which has the form $\sigma_{bf}(A_1, A_2) = \frac{c}{v}\sigma_{0bf}(A_1, A_2)$, where v is the relative velocity of the two atoms, c is the velocity of light (in vacuum).

When calculating σ_{0bf}, it is reasonable to use the $e^{i\frac{m_e}{(A_1-1)m_0}\mathbf{k}_1\cdot\mathbf{x}_e} = 1$ long wavelength approximation in the electron part of the Coulomb matrix element since $\frac{m_e}{(A_1-1)m_0} \ll 1$. The analysis of σ_{bf} shows that those processes give essential contribution to the cross section in which $k_e \ll k_1$ and $k_e \ll k_2$ where k_e, k_1 and k_2 are the magnitudes of the wave vectors of $\mathbf{k}_e$, $\mathbf{k}_1$ and $\mathbf{k}_2$. In this case as a consequence of momentum conservation the integration over $\mathbf{k}_1$ can be carried out resulting the substitution $\mathbf{k}_1 = -\mathbf{k}_2$. Thus E_f, which is the sum of the kinetic energies of the free electron, particle ${}^{A_1-1}_{Z_1}X$ and particle ${}^{A_2+1}_{Z_2}Y$, (in the Dirac-delta) and $E(k_e, k_2)$, which is the sum of the kinetic energies in the intermediate state (in the energy denominator) become $E_f = \hbar^2 k_2^2/[2m_0 a_{12}] + \hbar^2 k_e^2/(2m_e)$ with $a_{12} = (A_1-1)(A_2+1)/(A_1+A_2)$ and $E(k_e, k_2) = \frac{A_1}{A_1-1}\hbar^2 k_2^2/(2m_0) + \hbar^2 k_e^2/(2m_e)$, respectively (the intermediate neutron has wave vector $-\mathbf{k}_2$). These simplifications result

$$\sigma_{0bf} = \left(\frac{2Z_1}{3A_1}V_0\right)^2 \frac{\alpha_f^2 \hbar c}{(2\pi)^3}\int\int\int \delta(E_f - \Delta_{Bi}) \times \frac{\left|\sum_{m=-1}^{m=1} I_{ef}^m(\mathbf{k}_e) I_1^m(\mathbf{k}_2)\right|^2 |I_2(\mathbf{k}_2)|^2 d\mathbf{k}_e d\mathbf{k}_2}{\left[E(k_e, k_2) + \Delta_n - \Delta_- + E_{Bi}\right]^2}, \quad (5)$$

where $I_{ef}^m(\mathbf{k}_e) = \int u_i(\mathbf{x}_e) Y_{1m}^*(\Omega_e) x_e^{-2} e^{-i\mathbf{k}_e\cdot\mathbf{x}_e} d\mathbf{x}_e$, $I_1^m(\mathbf{k}_2) = \int \phi_i(\mathbf{x}_1) x_1 Y_{1m}(\Omega_{n1}) e^{-i\mathbf{k}_2\frac{A_1}{(A_1-1)}\cdot\mathbf{x}_1} d\mathbf{x}_1$, and $I_2(\mathbf{k}_2) = \int_{\Delta V} \phi_f^*(\mathbf{x}_2) e^{i\mathbf{k}_2\cdot\mathbf{x}_2} d\mathbf{x}_2$. Here $u_i(\mathbf{x}_e)$ and $\phi_i(\mathbf{x}_1)$ are the initial bound electron and neutron states and $\phi_f(\mathbf{x}_2)$ is the final bound neutron state. α_f denotes the fine structure constant and $\hbar$ is the reduced Planck-constant. $\Delta_n = 8.071\ MeV$ is the energy excess of the neutron. Furthermore, m_e is the rest mass of the electron and $\Delta_{Bi} = \Delta - E_{Bi}$. ΔV in $I_2(\mathbf{k}_2)$ is that volume of the surface of the second nucleus (of A_2) in which direct neutron capture may be assumed [13]. It can be considered as a shell of a sphere of radius R_{A_2} and of thickness L, where L is the mean free path of the ingoing neutron in the nucleus.

IV. CROSS SECTION OF CICP IN SPHERICAL NUCLEAR SHELL MODEL

The bound, initial electron, the initial and final nuclear states, which are used, have the form: $u_i(\mathbf{x}_e) = R_i(x_e) Y_{js}(\Omega_e)$, $\phi_i(\mathbf{x}_1) = \varphi_i(x_1) Y_{l_i m_i}(\Omega_1)/x_1$ and $\phi_f(\mathbf{x}_2) = \varphi_f(x_2) Y_{l_f m_f}(\Omega_2)/x_2$. Here j and s are orbital angular momentum and magnetic quantum numbers of the bound electron state. $\varphi_i(x_1)/x_1$ and $\varphi_f(x_2)/x_2$ are the radial parts of the one particle shell-model solutions of quantum numbers l_i, m_i and l_f, m_f. In the cases to be investigated the corresponding part $R_{0\Lambda} = b_k^{-1/2}\Gamma(\Lambda+3/2)^{-1/2}2^{1/2}\rho_k^{\Lambda+1}\exp(-\rho_k^2/2)$ of 0Λ one particle spherical shell model states [14] is applied as $\varphi_i(x_1)$ and $\varphi_f(x_2)$. Here $\rho_k = x_k/b_k$, $b_k = \left(\frac{\hbar}{m_0\omega_{sh,k}}\right)^{1/2}$ and $\hbar\omega_{sh,k} = 41A_k^{-1/3}$ (in MeV units, [11]) with $k = 1, 2$ corresponding to A_1 and A_2, and $\Gamma(x)$ is the gamma function.

Thus in the case of spherical shell model states

$$\sigma_{0bf,sh} = \frac{32}{3}\left(\frac{Z_1}{A_1}V_0\right)^2 (2l_f+1)\int \left|J_2^{l_f}(k_2)\right|^2 \times \int\sum_{l,\lambda} \frac{N_{l\lambda}\left|J_1^\lambda(k_2)\right|^2 \left|J_e^l(k_e)\right|^2}{\left[E(k_e, k_2) + \Delta_n - \Delta_- + E_{Bi}\right]^2} \times \alpha_f^2 \hbar c\,\delta(E_{f2} - \Delta_{Bi}) k_e^2 dk_e k_2^2 dk_2, \quad (6)$$

where in case of $0l_f$ final nuclear state with $\rho_f = R_{A_2}/b_2$

$$\left|J_2^{l_f}(k_2)\right|^2 = \frac{\pi L^2 \rho_f^{2l_f+3} e^{-\rho_f^2} J_{l_f+\frac{1}{2}}^2(k_2 R_{A_2})}{\Gamma\left(l_f+\frac{3}{2}\right) k_2}. \quad (7)$$

In the case of $0l_i$ initial nuclear state

$$J_1^\lambda(k_2) = \int R_{0l_i}(x_1)\, j_\lambda\left(\frac{A_1}{A_1-1}k_2 x_1\right) x_1^2 dx_1, \quad (8)$$

$$J_e^l(k_e) = F_{Cb}^{1/2}(k_e)\int R_i(x_e)\, j_l(k_e x_e) dx_e \quad \text{and} \quad (9)$$

$$N_{l\lambda} = (2l+1)(2\lambda+1)\begin{pmatrix} j & l & 1 \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} l_i & 1 & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2. \quad (10)$$

The parenthesized expressions are Wigner 3j symbols. (The suffix sh of any quantity denotes that it is calculated in the one particle spherical shell model.) In (9) $F_{Cb}(k_e)$is the Coulomb factor.

We restrict ourselves to $1s$ initial electronic state of $R_i(x_e) = 2a^{-3/2}\exp(-x_e/a)$ with $a = a_0/Z_{eff}$, where a_0 is the Bohr-radius, $Z_{eff} = \sqrt{E_B/Ry}$ and Ry is the Rydberg energy and use the $F_{Cb}(k_e) = 2\pi/(k_e a)$ approximation. Since $j = 0$ and $(2l+1)\begin{pmatrix} 0 & 1 & l \\ 0 & 0 & 0 \end{pmatrix}^2 = \delta_{1,l}$,

$$N_{1\lambda} = (2\lambda+1)\begin{pmatrix} l_i & 1 & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2. \quad (11)$$

Keeping the leading term of $J_e^1(k_e)$, introducing $k_2 = k_0 x$, and carrying out integration over k_e with the aid of the energy-Dirac-delta and in the case of $l_i = even$ $[l_i = 2;\ Ne(3/2^+, 0d)]$ to be investigated one obtains

$$\sigma_{0bf,sh} = \frac{2^{10}\pi^3}{3}\frac{Z_1^2 V_0^2}{A_1^2(\hbar c)^2}\frac{b_1^5 L^2}{\lambda\!\!\!^{-}_e a_0^2}\frac{m_0}{m_e}a_{12}(2l_f+1) \times \frac{\rho_f^{2l_f+3}e^{-\rho_f^2}}{\Gamma\left(l_f+\frac{1}{2}\right)}\sum_{\lambda=l_i\pm1}\frac{N_{1\lambda}(k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda+\frac{3}{2}\right)}S_\lambda. \quad (12)$$

Here $k_0 = \sqrt{2m_0c^2 a_{12}\Delta_B}/(\hbar c)$, $\lambda\!\!\!^{-}_e = \hbar/(m_e c)$,

$$S_\lambda = \int_0^1 f(x)\, g_\lambda(x)\, dx, \text{ with } x = k_2/k_0, \quad (13)$$

$$f(x) = \frac{\left(1-x^2\right)x^{2\lambda+1}e^{-(k_0 b_1)^2 x^2}J^2_{l_f+\frac{1}{2}}(xk_0R_{A_2})}{\left[1+\frac{\Delta_{Bi}}{E_B}(1-x^2)\right]^2\left[\frac{A_1 a_{12}}{A_1-1}x^2+1+\xi\right]^2}, \quad (14)$$

and $\xi = (\Delta_n - \Delta_- + E_{Bi})/\Delta_{Bi}$. In Eq.(13) $g_\lambda(x) = 1$ if $\lambda = l_i + 1$ and

$$g_\lambda(x) = (2l_i+1)^2 - 2(2l_i+1)(k_0 b_1 x)^2 + (k_0 b_1 x)^4 \quad (15)$$

if $\lambda = l_i - 1$.

The differential cross section $d\sigma_{0bf,sh}/dE_2$ of the process can be determined with the aid of

$$P(x) = \sum_{\lambda=l_i\pm1}\frac{N_{1\lambda}(k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda+\frac{3}{2}\right)}\frac{f(x)\, g_\lambda(x)}{x} \quad (16)$$

as $d\sigma_{0bf,sh}/dE_2 = K_{bf}\left[P(x)\right]_{x=\sqrt{z}}/(2E_{20})$ where $z = E_2/E_{20}$ with $E_{20} = (A_1-1)\Delta_{Bi}/(A_1+A_2)$, which is the possible maximum of the kinetic energy E_2 of particle ${}^{A_2+1}_{Z_2}Y$ (particle 5) created in the process, K_{bf} stands for the whole factor which multiplies the sum in (12). In the case of $e+\ {}^{21}_{10}Ne+\ {}^{21}_{10}Ne \to e'+\ {}^{20}_{10}Ne+\ {}^{22}_{10}Ne+\Delta$ reaction happening from the K shell, $l_i = l_f = 2$, $\Delta = 3.603\ MeV$, $E_{Bi} = 870.1\ eV$ and $\Delta_- = 1.310\ MeV$ ($2E_{20} = 3.26\ MeV$ and $K_{bf}/(2E_{20}) = 1.59\times10^{-35}\ cm^2 MeV^{-1}$). $d\sigma_{0bf,sh}/dE_2$ has accountable values near below $z = 1$, i.e. if $E_2 \sim E_{20}$.

The differential cross section $d\sigma_{0bf,sh}/dE_e = K_{bf}\left[P(x)\right]_{x=\sqrt{1-z}}/(2\Delta_{Bi})$ can also be determined with the aid of $P(x)$ where $z = E_e/\Delta_{Bi}$, E_e is the kinetic energy of the electron and K_{bf} is defined and is given above. $d\sigma_{0bf,sh}/dE_e$ has accountable values near above $z = 0$, i.e. if $E_e \sim 0$.

V. NUMERICAL CALCULATION IN CASE OF Ne

The transition probability per unit time λ_1 of CICP of one nucleus of mass number A_1 of an atomic gas of number density n created by all those isotopes of mass number A_2 for which CICP is allowed, reads in the spherical shell model as $\lambda_1 = cn\sum_{A_2} r_{A_2}\sigma_{0bf,sh}$ in the $\sigma = \sigma_{bf,sh}$ approximation used. Here r_{A_2} is the relative natural abundance of isotope of mass number A_2 (in the case of Ne to be investigated the $A_2 = A_1$ event is only possible). Furthermore $\tau_{1/2,1} = \ln 2/\lambda_1$, which can be considered as a half-life.

Finally, $r_{A_1}n\lambda_1$ is the rate per unit volume of the sample produced by the nuclei of mass number A_1 which take part as initial nuclei in CICP, and $r_{tot} = \sum_{A_1} r_{A_1}n\lambda_1$, which is the total rate per unit volume of the sample.

In the numerical calculation $V_0 = 50\ MeV$ is used [11]. In the case of Ne only the $e+\ {}^{21}_{10}Ne+\ {}^{21}_{10}Ne \to e'+\ {}^{20}_{10}Ne+\ {}^{22}_{10}Ne+\Delta$ reaction of $\Delta = 3.603\ MeV$ is allowed. CICP does not work in Ar since all the possible channels are energetically forbidden. On the other hand in the case of Kr and Xe nuclei the applicability of the spherical shell model may already be questionable. For Ne the transition probability per unit time is estimated as $\lambda_1 > \lambda_1(K)$, which is the transition probability per unit time of the bound-free CICP from the K shell of Ne. The initial and final states of ${}^{21}Ne$ and ${}^{22}Ne$ are supposed to be $0d$ spherical shell model states of $l_i = l_f = 2$, $r_{A_1} = r_{A_2} = r_{21} = 0.0027$, and $\Delta_- = 1.310\ MeV$. The electron binding energy in the K shell is $E_{Bi} = 870.1\ eV$. $\sigma_{0bf,sh}(K) = 8.25\times10^{-45}\ cm^{-2}$ is obtained in the case of bound-free CICP from the K shell of Ne. Taking this, the $\lambda_1 > 1.8\times10^{-17}\ s^{-1}$ and $\tau_{1/2,1} < 3.8\times10^{16}\ s$ ($1.2\times10^9\ y$) and $r_{tot} > 1.25\ cm^{-3}s^{-1}$ for a gas of normal state $\left(n = 2.652\times10^{19}cm^{-3},\ T = 273.15\ K,\ p = 100\ kPa\right)$. The estimated half life is so long that the decay through CICP does not alter natural abundance of Ne in observable measure. However, the rate is high enough to be measurable. ${}^{20}_{10}Ne$ and ${}^{22}_{10}Ne$ are mostly formed with energy near below $E_{10} = \frac{A_1+1}{A_1+A_2}\Delta_{Bi} = 1.97\ MeV$ and $E_{10} = \frac{A_1-1}{A_1+A_2}\Delta_{Bi} = 1.63\ MeV$ and with wave vectors of opposite direction. Therefore it is plausible to observe their creation in coincidence measurement.

VI. OTHER RESULTS AND DISCUSSION

Although the obtained λ_1 of Ne is rather small, the Weisskopf-estimation of the cross section indicate that it, and consequently λ_1, may be increased very much with the increase of the atomic number.

Moreover, the magnitudes of σ_{0bf} and λ_1 are very sensitive to the model applied, e.g. if neutron capture of nucleus of A_2 is not restricted to the direct reaction then the integral in $I_2(\mathbf{k}_2)$ must be carried out over the whole volume of the nucleus. In this case $\sigma_{0bf,sh}$ and λ_1 are increased by a factor of about 240. Furthermore, λ_1 can essentially increase e.g. with the increase of pressure. Therefore, contrary to the smallness of the value of λ_1 obtained in the case of Ne, CICP could play a role in the problem of nuclear waste disposal. In Table I. some long lived fission products are listed which can take part in CICP. The positive values of Δ_- and Δ_+ indicate that

isotope	$\tau(y)$	$A-1, A+1$	$\Delta_-(MeV)$	$\Delta_+(MeV)$
^{113m}Cd	14.1	112, 114	1.795	1.235
^{121m}Sn	55	120, 122	1.907	0.749
^{151}Sm	90	150, 152	2.475	0.186
^{79}Se	6.5×10^4	78, 80	1.108	1.842
^{93}Zr	1.53×10^6	92, 94	1.337	0.149
^{107}Pd	6.5×10^6	106, 108	1.533	1.149

TABLE I: Data for cooperative internal conversion process of long lived nuclear fission products. (Data to reaction (1).) $A-1$ and $A+1$ are the mass numbers of the two final izotopes, τ is the half-life of the fission product in y units. For the definition of Δ_- and Δ_+ see the text.

isotope	r_A	$\Delta_+(A)(MeV)$	$\Delta(MeV)$
$^{9}_{4}Be$	1.0	-1.259	4.587
$^{10}_{5}B$	0.199	3.382	9.228
$^{11}_{5}B$	0.812	-4.701	1.145
$^{12}_{6}C$	0.989	-3.125	2.721
$^{13}_{6}C$	0.011	0.105	5.951
$^{14}_{7}N$	0.99634	2.762	8.608
$^{15}_{7}N$	0.00366	-5.581	0.265
$^{16}_{8}O$	0.99762	-3.928	1.918
$^{17}_{8}O$	0.00038	-0.027	5.819

TABLE II: The values of the quantities $\Delta_+(A)$ and $\Delta = \Delta_-(dp)+\Delta_+(A)$ of the $e+d+ {}^{A}_{Z}X \to e'+p+ {}^{A+1}_{Z}X+\Delta$ reaction. r_A is the natural abundance of the isotope. $\Delta_-(dp) = \Delta_d - \Delta_p = 5.846\ MeV$, Δ_d and Δ_p are mass excesses of d and p.

each pair of the listed isotopes can produce CICP. Consequently, it seems to stand a practical chance to accelerate the decay of the listed isotopes if they are collected in appropriately high concentration and density in atomic state, which is, however, a great technical challenge.

Finally a special family of CICP reactions the $e + d + {}^{A}_{Z}X \to e' + p + {}^{A+1}_{Z}X + \Delta$ reaction family is worth mentioning. The quantity $\Delta_-(dp) = \Delta_d - \Delta_p = 5.846\ MeV$, which is characteristic of neutron loss of the deuteron. Here Δ_d and Δ_p are mass excesses of d and p. The energy of the reaction is $\Delta = \Delta_-(dp) + \Delta_+$ and some concrete reactions together with their Δ_+ and Δ values are listed, without completeness, in Table II.. Chances are that these reactions may open new perspectives in the field of nuclear energy production.

On the grounds of our results it can be stated that CICP seems to be able to significantly modify nuclear processes by surroundings.

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