Cooperative internal conversion process by proton exchange

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A generalization of the recently discovered cooperative internal conversion process is investigated theoretically. In the cooperative internal conversion process by proton exchange investigated the coupling of bound-free electron and proton transitions due to the dipole term of their Coulomb interaction permits cooperation of two nuclei leading to proton exchange and an electron emission. General expression of the cross section of the process obtained in the one particle spherical nuclear shell model is presented. As a numerical example the cooperative internal conversion process by proton exchange in *Al* is dealt with. As a further generalization, cooperative internal conversion process by heavy charged particle exchange and as an example of it the cooperative internal conversion process by triton exchange is discussed. The process is also connected to the field of nuclear waste disposal.

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In a recent paper [1] a new phenomenon, the cooperative internal conversion process (CICP) is discussed which is a special type of the well known internal conversion process [2]. In CICP two nuclei cooperate by neutron exchange creating final nuclei of energy lower than the energy of the initial nuclei. The process is initiated by the coupling of bound-free electron and neutron transitions due to the dipole term of their Coulomb interaction in the initial atom leading to the creation of a virtual free neutron which is captured through strong interaction by an other nucleus. The process can be written as

$$e + {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \to e' + {}^{A_1-1}_{Z_1}X + {}^{A_2+1}_{Z_2}Y + \Delta, \quad (1)$$

where ${}^{A_1}_{Z_1}X$, ${}^{A_2}_{Z_2}Y$ and ${}^{A_1-1}_{Z_1}X$, ${}^{A_2+1}_{Z_2}Y$ are the initial and final nuclei, respectively, and e is an initial, bound electron of atom containing ${}^{A_1}_{Z_1}X$ nucleus, and e' is a free, final electron. Δ is the energy of the reaction. $\Delta = \Delta_- + \Delta_+$, with $\Delta_- = \Delta_{A_1} - \Delta_{A_1-1}$ and $\Delta_+ = \Delta_{A_2} - \Delta_{A_2+1}$. Δ_{A_1}, Δ_{A_1-1}, Δ_{A_2}, Δ_{A_2+1} are the energy excesses of neutral atoms of mass numbers A_1, $A_1 - 1$, A_2, $A_2 + 1$, respectively [3]. The process is mainly related to atomic state, i.e. it has accountable cross section if the initial nuclei are located in free atoms therefore the cross section was determined in the case of free atoms (e.g. noble gases) in one particle nuclear and spherical shell models.

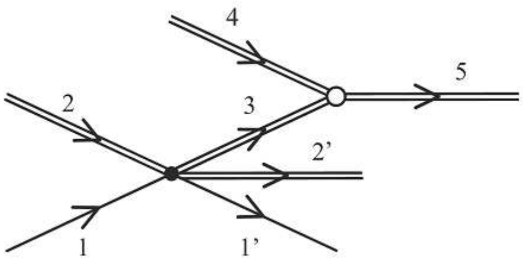

FIG. 1: The graph of cooperative internal conversion process by heavy charged particle (e.g. proton) exchange. Particle 1 (bound) and 1'(free) are electrons, particle 2 is the nucleus which looses the heavy charged particle (e.g. proton) and becomes particle 2'. Particle 3 is the intermediate heavy charged particle (e.g. proton). Particle 4 is the nucleus which absorbs the heavy charged particle (e.g. proton) and becomes particle 5. The filled dot denotes (in case of proton the dipole term of) the Coulomb-interaction and the open circle denotes nuclear (strong) interaction.

In this work the process

$$e_1 + {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \to e_1' + {}^{A_1-1}_{Z_1-1}V + {}^{A_2+1}_{Z_2+1}W + \Delta, \quad (2)$$

called cooperative internal conversion process by proton exchange (CICP-PE) is discussed theoretically in more detail. In this process (see Fig. 1) a bound proton of an atomic nucleus $\left({}^{A_1}_{Z_1}X, \text{ particle } 2\right)$ is virtually excited into a free state (particle 3) due to the dipole term V_{Cb}^{dip} of its Coulomb-interaction (in electric dipole coupling the proton has effective charge $q_p = (1 - Z_1/A_1)\, e$ [4]) with one of the bound atomic electrons (e_1) of the atom containing the ${}^{A_1}_{Z_1}X$ nucleus while the electron becomes free (e_1'). The free, virtual proton is captured by an other nucleus ${}^{A_2}_{Z_2}Y$ (particle 4) due to its nuclear potential V_{st} (created by strong interaction) forming the nucleus ${}^{A_2+1}_{Z_2+1}W$ (particle 5) in this way. The sum of the rest energies of the initial nuclei is E_{0i} and the sum of the the rest energies of the final nuclei is E_{0f} . If $E_{0i} - E_{0f} = \Delta > 0$, i.e. if $E_{0i} > E_{0f}$, then the process is energetically allowed and proton exchange is possible. The nuclear energy differ-

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ence Δ, which is the reaction energy, is shared between the kinetic energies of the final, free electron and the two final nuclei [${}^{A_1-1}_{Z_1-1}V$ (particle 2′) is the nucleus which has lost the proton].

The transition probability per unit time and the cross section $\sigma_{bf}(A_1, A_2)$ of CICP-PE with bound-free (bf) electron transitions can be determined with the aid of standard second order perturbation calculation of quantum mechanics. The cross section has the form $\sigma_{bf}(A_1, A_2) = \frac{c}{v}\sigma_{0bf}(A_1, A_2)$, where v is the relative velocity of the two atoms, c is the velocity of light (in vacuum). (The cross section of CICP-PE with bound-bound electron transition is neglected since it has proved to be much smaller than σ_{bf}.) The calculation of σ_{0bf} is similar to the calculation of CICP by neutron exchange [1]. The differences are the appearance of two Coulomb factors $F_{2'3}$ and F_{34} which multiply the cross section and Δ_n, $\frac{Z_1}{A_1}$ are changed to Δ_p, $\left(1-\frac{Z_1}{A_1}\right)$, respectively, in Eqs.(9)-(12) of [1]. $\Delta_p = 7.288969$ MeV is the energy excess of the proton. ($F_{2'3}$ and F_{34} are determined in the Appendix.) Here we repeat some essential features of the calculation. It is carried out in one particle nuclear model. The motions of the centers of mass of the two atoms are taken into account. Hydrogen like state of binding energy E_{Bi} and Coulomb-factor corrected plane wave are used as initial, bound and final, free electron states. The dipole term of the Coulomb interaction reads as $V_{Cb}^{dip} = \left(1-\frac{Z_1}{A_1}\right) e^2 \frac{4\pi}{3} x_1 x_e^{-2} \sum_{m=-1}^{m=1} Y_{1m}^*(\Omega_e) Y_{1m}(\Omega_1)$, where Z_1 and A_1 are charge and mass numbers of the first nucleus, e is the elementary charge, x_1, x_e and Ω_1, Ω_e are magnitudes and solid angles of vectors $\mathbf{x}_1$, $\mathbf{x}_e$ which are the relative coordinates of the proton and the electron in the first atom, respectively and Y_{1m} denotes spherical harmonics. (The order of magnitude of the cross section produced by the L-th pole coupling is $(R/r)^{2L-2}$ times smaller than the cross section produced by the dipole coupling where R and r are the nuclear and atomic radii. Therefore the leading term to the cross section is produced by the dipole coupling.) The motion of the intermediate proton and the two final nuclei are also described by plane waves. The rest masses of the two initial nuclei of mass numbers A_1 and A_2 are $m_1 = A_1 m_0$ (particle 2) and $m_2 = A_2 m_0$ (particle 4) where $m_0 c^2 = 931.494$ MeV is the atomic energy unit. For the nuclear potential a rectangular potential well is assumed, i.e. $V_{st} = -V_0$ ($x_2 \leq R_{A_2}$) and $V_{st} = 0$ ($x_2 > R_{A_2}$) where x_2 is the magnitude of vector $\mathbf{x}_2$, which is the relative coordinate of the neutron in the second nucleus and R_{A_2} is its radius. Direct proton capture may be assumed at the surface of the second nucleus (of A_2). The effective volume in which strong interaction induces proton capture can be considered as a shell of a sphere of radius R_{A_2} and of thickness L, where L is the mean free path of the ingoing proton in the nucleus [5].

Introducing the wave vectors $\mathbf{k}_e$ and $\mathbf{k}_1$, $\mathbf{k}_2$ of the free electron and particles ${}^{A_1-1}_{Z_1-1}V$ (particle 2′) and ${}^{A_2+1}_{Z_2+1}W$ (particle 5), respectively, the analysis of σ_{bf} shows that, similarly to the CICP by neutron exchange [1], those processes give essential contribution to the cross section in which $k_e \ll k_1$ and $k_e \ll k_2$ where k_e, k_1 and k_2 are the magnitudes of the wave vectors of $\mathbf{k}_e$, $\mathbf{k}_1$ and $\mathbf{k}_2$. (In this case as a consequence of momentum conservation $\mathbf{k}_1 = -\mathbf{k}_2$, furthermore the intermediate proton has wave vector $-\mathbf{k}_2$.)

The initial and final nuclear states have the form: $\phi_i(\mathbf{x}_1) = \varphi_i(x_1) Y_{l_i m_i}(\Omega_1)/x_1$ and $\phi_f(\mathbf{x}_2) = \varphi_f(x_2) Y_{l_f m_f}(\Omega_2)/x_2$ with $\varphi_i(x_1)/x_1$ and $\varphi_f(x_2)/x_2$ denoting the radial parts of the one particle shell-model solutions of quantum numbers l_i, m_i and l_f, m_f. For $\varphi_i(x_1)$ and $\varphi_f(x_2)$ the corresponding part $R_{0\Lambda} = b_k^{-1/2} \Gamma(\Lambda + 3/2)^{-1/2} 2^{1/2} \rho_k^{\Lambda+1} \exp(-\rho_k^2/2)$ of the 0Λ one particle spherical shell model states [6] is applied. Here $\rho_k = x_k/b_k$, $b_k = \left(\frac{\hbar}{m_0 \omega_{sh,k}}\right)^{1/2}$ and $\hbar\omega_{sh,k} = 41 A_k^{-1/3}$ (in MeV units, [4]) with $k = 1, 2$ corresponding to A_1 and A_2, and $\Gamma(x)$ is the gamma function. The case of spherical shell model states of $0l_i$ initial nuclear state and of $0l_f$ final nuclear state is investigated.

The initial electronic state is a $1s$ state of the form $R_i(x_e) = 2a^{-3/2} \exp(-x_e/a)$ with $a = a_0/Z_{eff}$, where a_0 is the Bohr-radius, $Z_{eff} = \sqrt{E_B/Ry}$ and Ry is the Rydberg energy. In the Coulomb-corrected plane wave applied for the final free electron the $F_{Cb}(k_e) = 2\pi/(k_e a)$ approximation is used, where $F_{Cb}(k_e)$ is the Coulomb factor of the electron. Keeping the leading term of $J_e^1(k_e)$ in [1] and in the case of $l_i = even$ [$l_i = 2$; $Al(5/2^+, 0d)$] to be investigated one obtains

$$\sigma_{0bf,sh} = \frac{2^{10}\pi^3}{3}\left(1-\frac{Z_1}{A_1}\right)^2 \frac{V_0^2}{(\hbar c)^2} \frac{b_1^5 L^2}{\lambda\!\!\!^{-}_e a_0^2} \frac{m_0}{m_e} a_{12} \times (2l_f+1) \frac{\rho_f^{2l_f+3} e^{-\rho_f^2}}{\Gamma\left(l_f+\frac{1}{2}\right)} \sum_{\lambda=l_i\pm1} \frac{N_{1\lambda}(k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda+\frac{3}{2}\right)} S_\lambda. \tag{3}$$

Here $\lambda\!\!\!^{-}_e = \hbar/(m_e c)$, m_e is the rest mass of the electron, $a_{12} = (A_1-1)(A_2+1)/(A_1+A_2)$, $\rho_f = R_{A_2}/b_2$, $k_0 = \sqrt{2m_0 \Delta_{Bi} a_{12}}/\hbar$, and

$$N_{1\lambda} = (2\lambda+1)\begin{pmatrix} l_i & 1 & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2. \tag{4}$$

The parenthesized expression is Wigner 3j symbol. (The suffix sh denotes that the quantity is calculated in the one particle spherical shell model.)

$$S_\lambda = \int_0^1 f(x)\, g_\lambda(x)\, h_1(x)\, h_2(x)\, dx, \tag{5}$$

$$f(x) = \frac{\left(1-x^2\right) x^{2\lambda+1} e^{-(k_0 b_1)^2 x^2} J_{l_f+\frac{1}{2}}^2(x k_0 R_{A_2})}{\left[1+\frac{\Delta_{Bi}}{E_B}\left(1-x^2\right)\right]^2 \left[\frac{A_1 a_{12}}{A_1-1} x^2 + 1 + \xi\right]^2}, \tag{6}$$

$x = k_2/k_0$, $\xi = (\Delta_p - \Delta_- + E_{Bi})/\Delta_{Bi}$ and $\Delta_{Bi} = \Delta - E_{Bi}$. $J_{l_f+\frac{1}{2}}$ is a Bessel-function of the first kind. In

Eq.(5) $g_\lambda(x) = 1$ if $\lambda = l_i + 1$ and

$$g_\lambda(x) = (2l_i + 1)^2 - 2(2l_i + 1)(k_0 b_1 x)^2 + (k_0 b_1 x)^4 \quad (7)$$

if $\lambda = l_i - 1$. $h_j(x) = d_j(x) / [\exp(d_j(x)) - 1]$, $j = 1, 2$ with

$$d_1(x) = 2\pi(Z_1 - 1)\alpha_f \frac{1}{x}\sqrt{\frac{(A_1 + A_2)\, m_0 c^2}{2A_1 (A_2 + 1)\, \Delta_{Bi}}} \quad (8)$$

and

$$d_2(x) = 2\pi Z_2 \alpha_f \frac{1}{x}\sqrt{\frac{(A_1 + A_2)\, m_0 c^2}{2(A_1 - 1)(A_2 + 1)\, \Delta_{Bi}}}. \quad (9)$$

α_f denotes the fine structure constant. In the numerical calculation $V_0 = 50\ MeV$ is used [4].

The differential cross section $d\sigma_{0bf,sh}/dE_2$ of the process can be determined with the aid of

$$P(x) = \sum_{\lambda = l_i \pm 1} \frac{N_{1\lambda} (k_0 b_1)^{2\lambda}}{\Gamma\left(\lambda + \frac{3}{2}\right)} \frac{f(x)\, g_\lambda(x)\, h_1(x)\, h_2(x)}{x} \quad (10)$$

as $d\sigma_{0bf,sh}/dE_2 = K_{bf}\, [P(x)]_{x=\sqrt{z}} / (2E_{20})$ where $z = E_2/E_{20}$ with $E_{20} = (A_1 - 1)\, \Delta_{Bi} / (A_1 + A_2)$, which is the possible maximum of the kinetic energy E_2 of particle ${}^{A_2+1}_{Z_2+1}W$ (particle 5) created in the process, K_{bf} stands for the whole factor which multiplies the sum in (3). $d\sigma_{0bf,sh}/dE_2$ has accountable values near below $z = 1$, i.e. if $E_2 \sim E_{20}$.

The differential cross section $d\sigma_{0bf,sh}/dE_e = K_{bf}\, [P(x)]_{x=\sqrt{1-z}} / (2\Delta_{Bi})$ can also be determined with the aid of $P(x)$ where $z = E_e/\Delta_{Bi}$, E_e is the kinetic energy of the electron and K_{bf} is defined above. $d\sigma_{0bf,sh}/dE_e$ has accountable values near above $z = 0$, i.e. if $E_e \sim 0$.

It is a special case of (2) if the two initial nuclei are identical. In this case the CICP-PE reads as

$$e_1 + {}^{A_1}_{Z_1}X + {}^{A_1}_{Z_1}X \to e_1' + {}^{A_1-1}_{Z_1-1}V + {}^{A_1+1}_{Z_1+1}W + \Delta. \quad (11)$$

For example of such a case the reaction $e + {}^{27}_{13}Al + {}^{27}_{13}Al \to e' + {}^{26}_{12}Mg + {}^{28}_{14}Si + \Delta$ is considered when the reaction starts from the K shell. The initial and final nuclear states are supposed to be $0d$ spherical shell model states of $l_i = l_f = 2$, $\Delta = 3.31362\ MeV$. The electron binding energy in the K shell is $E_{Bi} = 1.5596\ keV$ and $\Delta_- = -0.98235\ MeV$. In this case $2E_{20} = 3.1894\ MeV$ and $K_{bf}/(2E_{20}) = 2.41 \times 10^{-35}\ cm^2 MeV^{-1}$). $\sigma_{0bf,sh}(K) = 2.41 \times 10^{-46}\ cm^{-2}$ is obtained in the case of bound-free CICP from the K shell of Al. If one compares this result with $\sigma_{0bf,sh}(K) = 8.25 \times 10^{-45}\ cm^{-2}$obtained in case of CICP by neutron exchange in Ne one can recognize that the ratio of the two cross sections is only 0.030. At first sight it seems to be larger when expected since two Coulomb factors appeare in the cross section. But as it was said earlier the intermediate proton has wave vector $-\mathbf{k}_2$ and thus its energy $E_3 = \hbar^2 \mathbf{k}_2^2 / (2m_0)$ with $\hbar k_2 = \hbar k_0 x = x\sqrt{2m_0 \Delta_{Bi} a_{12}}$. It gives $E_3 = x^2 \Delta_{Bi} A_1/2$ since $a_{12} = A_1/2$ if $A_1 = A_2$ and near below $x = 1$ the value of E_3 is large enough to result moderately small Coulomb factors.

$Isotope$	$Products$	$\Delta_-(MeV)$	$\Delta_+(MeV)$	$\Delta(MeV)$
${}^{19}F$	${}^{18}O, {}^{20}Ne$	−0.705	5.555	4.850
${}^{23}Na$	${}^{22}Ne, {}^{24}Mg$	−1.505	4.404	2.899
${}^{27}Al$	${}^{26}Mg, {}^{28}Si$	−0.982	4.296	3.314
${}^{31}P$	${}^{30}Si, {}^{32}S$	−0.008	1.575	1.567
${}^{45}Sc$	${}^{44}Ca, {}^{46}Ti$	0.400	3.056	3.456
${}^{55}Mn$	${}^{54}Cr, {}^{56}Fe$	−0.778	2.895	2.117
${}^{59}Co$	${}^{58}Fe, {}^{60}Ni$	−0.075	2.245	2.170
${}^{103}Rh$	${}^{102}Ru, {}^{104}Pd$	1.076	1.369	2.445
${}^{127}I$	${}^{126}Te, {}^{128}Xe$	1.083	0.873	1.956
${}^{133}Cs$	${}^{132}Xe, {}^{134}Ba$	1.204	0.879	2.083

TABLE I: Data for cooperative internal conversion process by proton exchange. (Data to reaction (11).) In the first column the initial stable isotope (of unity relative natural abundance) and in the second column the reaction products can be found. For the definition of Δ_-, Δ_+ and Δ see the text.

For a gas of atomic Al and of number density n the transition probability per unit time $\lambda_1 = cn \sum_{A_2} r_{A_2} \sigma_{0bf,sh} = cn\sigma_{0bf,sh}$ [1] since the relative natural abundance r_{A_2} of the initial ${}^{27}_{13}Al$ isotope equals unity. λ_1 is estimated as $\lambda_1 > \lambda_1(K)$, which is the transition probability per unit time of the bound-free CICP-PE from the K shell of Al ($\lambda_1(K) = cn\sigma_{0bf,sh}(K)$), resulting $\lambda_1 > 1.92 \times 10^{-16}\ s^{-1}$ and $r_{tot} > 5.09 \times 10^3\ cm^{-3}s^{-1}$ for a gas of normal state ($n = 2.652 \times 10^{19} cm^{-3}$, $T = 273.15$ K, $p = 100\ kPa$ and $r_{tot} = n\lambda_1$, which is the total rate per unit volume of the sample, in this case since $r_{A_1} = 1$ [1]).

In Table I. the Δ_-, Δ_+ and Δ data of some cooperative internal conversion processes by proton exchange (data to reaction (11) can be found. In the first column the initial stable isotope of relative natural abundance unity and in the second column the reaction products are listed.

There are other possibilities to realize CICP, when a charged heavy particle (such as d, t, ${}^3_2He$ and ${}^4_2He$) is exchanged instead of proton exchange. The process is called cooperative internal conversion process by heavy charged particle exchange (CICP-HCPE) and it can be visualized with the aid of Fig.1 too. Denoting the intermediate particle (particle 3 in Fig. 1) by ${}^{A_3}_{z_3}w$, which is exchanged, the cooperative internal conversion process by heavy charged particle exchange reads as

$$e + {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \to e' + {}^{A_1-A_3}_{Z_1-z_3}V + {}^{A_2+A_3}_{Z_2+z_3}W + \Delta. \quad (12)$$

Here e and e' denote bound and free electron and Δ is the energy of the reaction, i.e. the difference between the rest energies of initial $\left({}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y\right)$ and final $\left({}^{A_1-A_3}_{Z_1-z_3}V + {}^{A_2+A_3}_{Z_2+z_3}W\right)$ states. $\Delta = \Delta_- + \Delta_+$, with $\Delta_- = \Delta^{A_1}_{Z_1} - \Delta^{A_1-A_3}_{Z_1-z_3}$ and $\Delta_+ = \Delta^{A_2}_{Z_2} - \Delta^{A_2+A_3}_{Z_2+z_3}$. $\Delta^{A_1}_{Z_1}$,

Isotope	*Products*	$\Delta_-(MeV)$	$\Delta_+(MeV)$	$\Delta(MeV)$
^{19}F	$^{16}O,^{22}Ne$	3.250	6.537	9.787
^{23}Na	$^{20}Ne,^{26}Mg$	−2.488	6.685	4.197
^{27}Al	$^{24}Mg,^{30}Si$	−3.263	7.236	3.973
^{31}P	$^{28}Si,^{34}S$	−2.948	5.491	2.543
^{45}Sc	$^{42}Ca,^{48}Ti$	−2.522	7.418	4.896
^{55}Mn	$^{52}Cr,^{58}Fe$	−2.294	4.443	2.149
^{59}Co	$^{56}Fe,^{62}Ni$	−1.623	4.519	2.896
^{103}Rh	$^{100}Ru,^{106}Pd$	1.197	1.882	3.079
^{127}I	$^{124}Te,^{130}Xe$	1.536	0.893	2.429
^{133}Cs	$^{130}Xe,^{136}Ba$	1.806	0.816	2.622

TABLE II: Data for cooperative internal conversion process by triton exchange. (Data to reaction (14).) In the first column the initial stable isotope (of unity relative natural abundance) and in the second column the reaction products can be found. For the definition of Δ_-, Δ_+ and Δ see the text.

$\Delta_{Z_1-z_3}^{A_1-A_3}$, $\Delta_{Z_2}^{A_2}$, $\Delta_{Z_2+z_3}^{A_2+A_3}$ are the energy excesses of neutral atoms of mass number-charge number pairs A_1, Z_1; A_1-A_3, Z_1-z_3; A_2, Z_2; A_2+A_3, Z_2+z_3, respectively [3].

In (12) the initial bound electron (particle 1) Coulomb interacts with the nucleus ${}_{Z_1}^{A_1}X$ (particle 2). A free electron (particle 1′), the intermediate particle ${}_{z_3}^{A_3}w$ (particle 3) and the nucleus ${}_{Z_1-z_3}^{A_1-A_3}V$ (particle 2′) are created due to this interaction. The intermediate particle ${}_{z_3}^{A_3}w$ (particle 3) is captured due to the strong interaction by the nucleus ${}_{Z_2}^{A_2}Y$ (particle 4) forming the nucleus ${}_{Z_2+z_3}^{A_2+A_3}W$ (particle 5) in this manner. So in (12) the nucleus ${}_{Z_1}^{A_1}X$ (particle 2) looses a particle ${}_{z_3}^{A_3}w$ which is taken up by the nucleus ${}_{Z_2}^{A_2}Y$ (particle 4). The process is energetically forbidden if $\Delta < 0$.

As an example we deal with the

$$e + {}_{Z_1}^{A_1}X + {}_{Z_2}^{A_2}Y \rightarrow e' + {}_{Z_1-1}^{A_1-3}V + {}_{Z_2+1}^{A_2+3}W + \Delta \quad (13)$$

reaction in which a triton is exchanged. It is called CICP by triton exchange (CICP-TE). Special type of reaction (13) is

$$e + {}_{Z_1}^{A_1}X + {}_{Z_1}^{A_1}X \rightarrow e' + {}_{Z_1-1}^{A_1-3}V + {}_{Z_2+1}^{A_2+3}W + \Delta. \quad (14)$$

In Table II. the Δ_-, Δ_- and Δ data of some cooperative internal conversion processes by triton exchange (data to reaction (14) can be found.

In Table III. some long lived fission products are listed which can take part in CICP-PE. The values of Δ_- and Δ_- indicate that each pair of the listed isotopes can produce CICP-PE since $\Delta_- + \Delta_- = \Delta > 0$ in every case. Consequently similarly to those long lived fission products which can decay through CICP by neutron exchange [1], it seems to stand also a practical chance to accelerate the decay of the listed isotopes if they were collected in appropriately high concentration and density in atomic state.

Isotope	$\tau(y)$	*Products*	$\Delta_-(MeV)$	$\Delta_+(MeV)$
^{99}Tc	2.11×10^5	$^{98}Mo,^{100}Ru$	0.789	1.896
^{129}I	1.57×10^7	$^{128}Te,^{130}Xe$	0.491	1.378
^{135}Cs	2.3×10^6	$^{134}Xe,^{136}Ba$	0.538	1.305
^{137}Cs	30.07	$^{136}Xe,^{138}Ba$	−0.126	1.716
^{155}Eu	4.7611	$^{154}Sm,^{156}Gd$	0.637	0.717

TABLE III: Data for cooperative internal conversion process by proton exchange of long lived nuclear fission products. (Data to reaction (11).) Products are the two stable final izotopes, τ is the half-life of the fission product in y units. For the definition of Δ_- and Δ_+ see the text.

A. Appendix - Coulomb factors $F_{2'3}$ and F_{34}

Since particles $2'$, 3 and 4 all have positive charge, furthermore they are all heavy, the two essential Coulomb factors, which appear in the cross section, are $F_{2'3}$ and F_{34}. Since Coulomb factors $F_{2'3}$ and F_{34} determine the order of magnitude of the cross section of the process (as it is proportional to $F_{2'3}F_{34}$) we treat them in more detail in the case of CICP-PE in the following.

We adopt the approach standard in nuclear physics when describing the cross section of nuclear reactions of heavy, charged particles j and k of like positive charge of charge numbers z_j and z_k and of relative kinetic energy E. The cross section of such a process can be derived applying the Coulomb solution $\varphi(\mathbf{r})$,

$$\varphi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}f(\mathbf{k},\mathbf{r})/\sqrt{V}, \quad (15)$$

which is the wave function of a free particle of charge number z_j in a repulsive Coulomb field of charge number z_k [7], in the description of relative motion of projectile and target. In (15) V denotes the volume of normalization, $\mathbf{r}$ is the relative coordinate of the two particles, $\mathbf{k}$ is the wave number vector in their relative motion and

$$f(\mathbf{k},\mathbf{r}) = e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})\,{}_1F_1(-i\eta_{jk},1;i[kr-\mathbf{k}\cdot\mathbf{r}]), \quad (16)$$

where ${}_1F_1$ is the confluent hypergeometric function and Γ is the Gamma function. Since $\varphi(\mathbf{r}) \sim e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})$, the cross section of the process is proportional to

$$\left|e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})\right|^2 = \frac{2\pi\eta_{jk}(E)}{\exp[2\pi\eta_{jk}(E)]-1} = F_{jk}(E), \quad (17)$$

which is the so-called Coulomb factor. Here

$$\eta_{jk}(E) = z_j z_k \alpha_f \sqrt{a_{jk}\frac{m_0c^2}{2E}} \quad (18)$$

is the Sommerfeld parameter in the case of colliding particles of mass numbers A_j, A_k and rest masses $m_j = A_j m_0$, $m_k = A_k m_0$. $m_0c^2 = 931.494\ MeV$ is the atomic energy unit, α_f is the fine structure constant and E is taken in the center of mass (CM) coordinate system.

$$a_{jk} = \frac{A_j A_k}{A_j + A_k} \quad (19)$$

is the reduced mass number of particles j and k of mass numbers A_j and A_k.

If initial particles have negligible initial momentum then in the final state $\mathbf{k}_1 = -\mathbf{k}_2$ ($\mathbf{k}_{particle,2'} = -\mathbf{k}_{particle,5}$) because of momentum conservation. (It was obtained [1] that the process has accountable cross section if the momentum of the final electron can be neglected.) In this case the momentum and energy of the virtual particle 3 (e.g. proton) are $\mathbf{k}_{particle,3} = -\mathbf{k}_{particle,2'} = \mathbf{k}_{particle,5} \equiv \mathbf{k}_2$ and $E_3 = \hbar^2 \mathbf{k}_2^2 / (2m_3)$, where $\hbar$ is the reduced Planck-constant. Calculating the Coulomb factor $F_{2'3}$ [see (17)] between particles $2'$ and 3 the energy E_3 is given in their CM coordinate system (since $\mathbf{k}_{particle,3} = -\mathbf{k}_{particle,2'}$) thus E_3 can be substituted directly in (18) producing

$$\eta_{2'3} = (Z_1 - 1)\,\alpha_f \frac{1}{x} \sqrt{\frac{A_1 + A_2}{A_1 (A_2 + 1)} \frac{m_0 c^2}{2\Delta_{Bi}}}. \qquad (20)$$

in case of proton exchange. Here the $k_2 = k_0 x$ substitution is also used. Calculating the Coulomb factor F_{34}, the energy E_3 of particle 3 is now given in the laboratory frame of reference since particle 4 is at rest. In the CM system of particles 3 and 4 the energy $E_3(CM)$ is

$$E_3(CM) = \frac{A_{particle,4}}{(A_{particle,3} + A_{particle,4})} E_3. \qquad (21)$$

Substituting it into (18)

$$\eta_{34} = Z_2 \alpha_f \frac{1}{x} \sqrt{\frac{(A_1 + A_2)\, m_0 c^2}{2\,(A_1 - 1)\,(A_2 + 1)\, \Delta_{Bi}}} \qquad (22)$$

in case of proton exchange.

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