Recoil assisted low energy nuclear reactions

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Low energy nuclear processes that are strongly hindered by Coulomb repulsion between the reacting nuclei, are investigated in solid environment. It is shown that the hindering effect may be significantly weakened (practically it diappears) if one takes into account the Coulomb interaction of one of the reacting particles with the surroundings. It is obtained that if the modification of the wave function due to Coulomb interaction with charged constituents of the environment is taken into account applying standard perturbation calculation of quantum mechanics then wave components of high momentum with small amplitude are mixed to the initial wave of small momentum. To these partial waves of high momentum much higher Coulomb factor can be attached that can drastically increase the cross section. The mechanism (called recoil assistance) opens the door to a great variety of nuclear processes that so far have been thought to have negligible rate at low energies. The recoil assisted nuclear *pd* reaction is investigated like a sample reaction numerically. Low energy nuclear reactions allowed by recoil assistance and leading to nuclear transmutations are partly overviewed. Critical analysis of Fleischmann-Pons type low energy nuclear reaction experiments is presented too.

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I. INTRODUCTION

Since the "cold fusion" publication by Fleischmann and Pons in 1989 [1] a new field of experimental physics has emerged. Although, even the existence of the phenomenon of nuclear fusion at low energies is in doubt in mainstream physics, the quest for low-energy nuclear reactions (LENR) is on and hundreds of publications (mostly experimental) have been devoted to various aspects of the problem. (For a summary of experimental observations, theoretical efforts and background events see e.g. [2], [3], [4].)

The main reasons for revulsion against the topic according to standard nuclear physics have been: (a) due to the Coulomb repulsion no nuclear reaction should take place at energies corresponding to room temperature, (b) the observed extra heat attributed to nuclear reactions is not accompanied by the nuclear end products expected from hot fusion experiences, (c) traces of nuclear transmutations were also observed, that considering the repulsive Coulomb interaction is an even more inexplicable fact at these energies. It is also against acceptance of the phenomenon that (d) its reproduction seems to be uncertain. It is evident that these serious questions must be at least qualitatively answered in order to transfer the field back into proper science.

On the other hand in the last two decades, investigating astrophysical factors of nuclear reactions of low atomic numbers, which have great importance in nuclear astrophysics [5], in the cross section measurements of the *dd* reactions in deuterated metal targets extraordinary observations, called anomalous screening, were made in low energy accelerator physics ([6] and references there in). Several years ago a systematic survey of the experimental methods applied in investigating and the theoretical efforts for the explanation of the anomalous screening effect was made [6] from which one can conclude that the full theoretical explanation of the effect is still missing.

Observations in the above two fields motivated us to search for physical phenomena that may have modifying effect on nuclear reactions in solid state environment. Earlier we theoretically found [7] that if the reaction $p + d \rightarrow {}^3He$ takes place in a solid material then the nuclear energy is mostly taken away by an electron of the environment instead of the emission of a γ photon, a result that calls the attention to the possible role of the presence of charged particles of the surroundings.

The cross section of direct (first order in terms of perturbation calculation) nuclear reactions has strongly and nonlinearly decreasing energy dependence due to the repulsive Coulomb potential in the low energy range (as will be seen in the next section). In second order processes, however, this huge decrease of the cross section may disappear. Therefore the cross section of a not direct (second order) reaction may be essentially higher than the cross section of the direct reaction. In this paper we attempt to demonstrate the effect.

We investigate the

$$ {}^{A_1}_{z_1}V + p + {}^{A_3}_{z_3}X \rightarrow {}^{A_1}_{z_1}V' + {}^{A_3+1}_{z_3+1}Y + \Delta \qquad (1)$$

process which we are going to call recoil assisted proton capture. The solid is a metal ${}^{A_1}_{z_1}V$ (e.g. Pd) irradiated with low energy free protons meanwhile free ions or atoms of nuclei ${}^{A_3}_{z_3}X$ (e.g. deuterons) are also present. The particles ${}^{A_1}_{z_1}V$ interact with the ingoing free protons via Coulomb interaction and push them (virtually) into an intermediate state of large momentum. These protons are captured by the nuclei ${}^{A_3}_{z_3}X$ due to strong interaction and a nucleus ${}^{A_3+1}_{z_3+1}Y$ is created in this manner. The particles ${}^{A_1}_{z_1}V$ (initial) and ${}^{A_1}_{z_1}V'$ (final) assist the process only. Δ is the energy of the reaction.

The recoil assisted $p+d\rightarrow{}^{3}He$ reaction

$$ {}_{z_1}^{A_1}V+p+d\rightarrow{}_{z_1}^{A_1}V'+{}_{2}^{3}He+\Delta \qquad (2)$$

is discussed in more detail. Here $\Delta=5.493\ MeV$.

The reaction

$$q+{}_{z_2}^{A_2}w+{}_{z_3}^{A_3}X\rightarrow q'+{}_{z_3+z_2}^{A_3+A_2}Y+\Delta, \qquad (3)$$

a generalization of (1), is also dealt with. Here q is any charged particle of the environment.

In Section II. the decreasing energy dependence of the cross section in the low energy range is connected to the hindering role of Coulomb repulsion and the Coulomb factor. In Section III. the basic ideas are expounded. Sections IV. and V. are devoted to the deduction of transition probability per unit time and cross section of recoil assisted $p+d\rightarrow{}^{3}He$ reaction in a very simple, illustrative model. In Section VI. some other recoil assisted low energy nuclear reactions, which may lead to nuclear transmutations, are overviewed. Sections VII. and VIII. are devoted to critical analysis of electrolysis experiments and summary. Some correspondences and identities applied in the calculation can be found in the Appendix (Section IX.).

II. HINDERING ROLE OF COULOMB REPULSION

It is standard in nuclear physics that heavy particles j and k of like positive charge need considerable amount of relative kinetic energy E determined by the height of the Coulomb barrier in order to let the probability of their nuclear interaction have significant value.

The cross section of such a process can be derived applying the Coulomb solution $\varphi(\mathbf{r})$,

$$\varphi(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}f(\mathbf{k},\mathbf{r})/\sqrt{V}, \qquad (4)$$

which is the wave function of a free particle of charge number z_j in a repulsive Coulomb field of charge number z_k [8], in the description of relative motion of projectile and target. In (4) V denotes the volume of normalization, $\mathbf{r}$ is the relative coordinate of the two particles, $\mathbf{k}$ is the wave number vector in their relative motion and $f(\mathbf{k},\mathbf{r})=e^{-\pi\eta_{jk}/2}\times\ \Gamma(1+i\eta_{jk})\times\ {}_1F_1(-i\eta_{jk},1;i[kr-\mathbf{k}\cdot\mathbf{r}])$, where ${}_1F_1$ is the confluent hypergeometric function and Γ is the Gamma function. $\varphi(\mathbf{r})\sim e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})$ and

$$\left|e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})\right|=\sqrt{\frac{2\pi\eta_{jk}\left(E\right)}{\exp\left[2\pi\eta_{jk}\left(E\right)\right]-1}}=f_{jk}(E), \qquad (5)$$

which is the so-called Coulomb factor. Here

$$\eta_{jk}\left(E\right)=z_jz_k\alpha_f\sqrt{a_{jk}\frac{m_0c^2}{2E}} \qquad (6)$$

is the Sommerfeld parameter.

$$a_{jk}=\frac{A_jA_k}{A_j+A_k} \qquad (7)$$

is the reduced mass number of colliding particles j and k of mass numbers A_j and A_k and rest masses $m_j=A_jm_0$, $m_k=A_km_0$. $m_0c^2=931.494\ MeV$ is the atomic energy unit, α_f is the fine structure constant and E is taken in the center of mass (CM) coordinate system.

The cross section of the process is proportional to $f_{jk}^2(E)$ thus the rate of the nuclear reaction of heavy, charged particles of like positive charge becomes very small at low energies in consequence of $f_{jk}^2(E)$ being very small. Mathematically it appears in the energy dependence of the cross section (σ) of the charged-particle induced reactions as

$$\sigma\left(E\right)=S\left(E\right)\exp\left[-2\pi\eta_{jk}\left(E\right)\right]/E, \qquad (8)$$

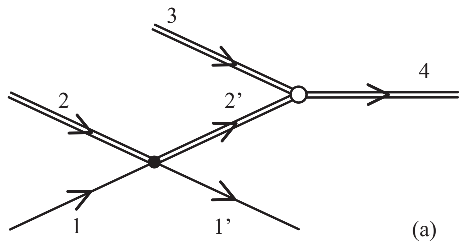
where $S\left(E\right)$ is the astrophysical factor, which can be written as $S(E)=S(0)+S_1E+S_2E^2$, [5].

It is the consequence of energy dependence (8) of the cross section that to this day it is a commonplace that the rate of any nuclear reaction between heavy, charged particles of positive charge is unobservable at low energies. The aim of this paper is to show that in solids, contrary to the above assumption, there can exist nuclear processes with observable rate at low energies.

It will be shown that the extremely huge increment in the Coulomb factor due to recoil makes it possible for the cross section of recoil assisted nuclear reactions to reach an observable magnitude even in the very low energy case. Since the actual Coulomb factors are the clue to the recoil assisted nuclear reactions we focus our attention on to them especially concerning the Coulomb factors of heavy charged particles.

III. GENERAL CONSIDERATIONS

It is assumed that the state of one participant of the nuclear process is changed by Coulomb interaction with a charged particle of the environment (e.g. solid material, a metal) before the nuclear process takes place. As an example, which will be considered in more detail, let us consider the $p+{}_{z_3}^{A_3}X\rightarrow{}_{z_4}^{A_4}Y$ (especially the $p+d\rightarrow{}_{2}^{3}He$) model-process $(A_4=A_3+1,\ z_4=z_3+1)$. In this reaction slow initial protons enter the solid (e.g. Pd) while quasi-free ${}_{z_3}^{A_3}X$ particles (e.g. deuterons) are also present. The graphs of the process can be seen in Fig. 1. In this case, one of the slow protons (as particle 2) can enter into Coulomb interaction with a heavy constituent (Pd atom) of the solid (as particle 1) knocking out it from the lattice before the nuclear reaction with an ${}_{z_3}^{A_3}X$ particle (e.g. a deuteron). The Coulomb interaction is followed by strong interaction, that induces a nuclear process e.g. a nuclear capture process, in which the energy Δ of the nuclear reaction is divided between



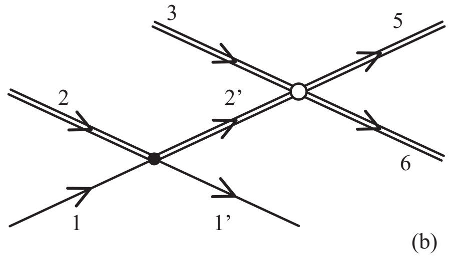


FIG. 1: The graphs of recoil assisted nuclear reactions. The simple lines represent (initial localized (1) and final free(1')) heavy constituent of the solid. The doubled lines represent free, heavy, charged initial (2) particles (such as p, d), their intermediate state (2'), target nuclei (3) and reaction products (4, 5, 6). The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction. FIG. 1(a) is a capture process and FIG. 1(b) is a reaction with two fragments.

the heavy constituent (Pd atom) and the heavy nuclear product ${}^{A_4}_{z_4}Y$ (e.g. ${}^{3}_{2}He$). This is a second order process in terms of standard perturbation theory (see FIG. 1(a)). The rest masses of the participants are: $m_1 = A_1 m_0$, $m_2 = m_0$, $m_3 = A_3 m_0$ (e.g. $m_3 = 2m_0$) and $m_4 = A_4 m_0$ (e.g. $m_4 = 3m_0$) corresponding to the heavy constituent, the proton, the other nucleus (e.g. deuteron) and the nuclear product ${}^{A_4}_{z_4}Y$ (e.g. ${}^{3}_{2}He$), respectively. The states of the heavy constituent, the proton, the other nucleus and the nuclear product must fulfill energy and momentum conservation in the initial and final states. If initially (before the Coulomb interaction) protons and ${}^{A_3}_{z_3}X$ particles (e.g. deuterons) move slowly and the heavy constituent has only vibration of thermal origin then their initial momenta can be neglected and the initial total momentum can be taken to be zero. Therefore in the final state $\mathbf{p}_{1'} + \mathbf{p}_4 = 0$, where $\mathbf{p}_{1'}$ and $\mathbf{p}_4$ are the momenta of the heavy constituent, knocked out from the lattice, and the nuclear product resulting $\mathbf{p}_{1'} = -\mathbf{p}_4 (= \mathbf{p})$. From energy conservation it follows that $\mathbf{p}^2/(2m_1) + \mathbf{p}^2/(2m_4) = \Delta$ which determines $p = |\mathbf{p}| = \sqrt{2m_0 a_{14}\Delta}$. (If $m_1 > m_4$ then the nuclear product (particle 4) will take away the larger part of the total nuclear reaction energy Δ.) If the Coulomb interaction preserves the momentum (as will be shown later) then the Coulomb scattered proton (particle 2') has momentum $\mathbf{p}_{2'} = \mathbf{p}_4 = -\mathbf{p}$ too. It means that its energy $E_{2'} = \frac{p^2}{2m_2} = a_{14}\Delta/A_2$ (e.g. $E_{2'} = \frac{3A_1}{A_1+3}\Delta$ in the case of $p + d \rightarrow {}^{3}_{2}He$ reaction) in the intermediate state. (A_1 is the nucleon number of the heavy constituent (Pd atom) and $A_2 = 1$ in the case of proton capture discussed.) The Coulomb factor must be calculated in the CM system. $E_{2'}(lab) = a_{14}\Delta/A_2$ is valid in the laboratory frame of reference (lab). Thus $E_{2'}(CM) = \frac{m_3}{m_2+m_3}E_{2'}(lab) = \frac{A_3}{A_2(A_2+A_3)}a_{14}\Delta$ must be substituted in (6) that results

$$\eta_{2'3} = z_2 z_3 \alpha_f A_2 \sqrt{\frac{m_0 c^2}{2a_{14}\Delta}} \tag{9}$$

and which gives $\eta_{2'3} = z_3\alpha_f\sqrt{m_0c^2/(2a_{14}\Delta)}$ with $z_2 = 1$ and $A_2 = 1$ corresponding to proton capture. In the case of $p + d \rightarrow {}^{3}_{2}He$ reaction $\Delta = 5.493$ MeV [9], which is the reaction energy and if $A_1 = 106$ corresponding to one of the Pd isotopes then $2a_{14}\Delta = 32.05$ MeV, $\eta_{2'3} = 0.039$ and $f^2_{2'3} = 0.88$.

Moreover, in a proton capture process with particle 3 of mass number A_3 the mass number of particle 4 becomes $A_4 = A_3 + 1$ and the reduced mass number $a_{14} = A_1(A_3+1)/(A_1+A_3+1)$. If $A_1 \simeq A_3 = \widehat{A}$ then $a_{14} = \widehat{A}/2$ and $2a_{14}\Delta = \widehat{A}\Delta = 572$ MeV at $\widehat{A} = 100$ and with a typical (averaged) value of $\Delta = 5.72$ MeV (see TABLE I.). If $z_3 = 46$ then $\eta_{2'3} = 0.428$ and $f^2_{2'3} = 0.196$.

Thus the Coulomb scattered proton will have a (virtual) momentum (a virtual kinetic energy) in the intermediate state that is large enough to make a drastic increase of the Coulomb factor $f_{2'3}$ as has been demonstrated above in the two numerical examples. As a conclusion, the state of an ingoing heavy, charged particle may be changed by a solid environment in such a way that must not be neglected in rate calculations of nuclear processes.

IV. MODEL OF RECOIL ASSISTED NUCLEAR REACTIONS IN d-FILLED METALS

The reaction of heavy charged particles induced by solid state environment (deuterized metals in a proton flux from an accelerator) is modelled in the following way. Let us take two independent systems A and B, where A is a solid and B is an ensemble of free, heavy charged particles (e.g. free protons) with the corresponding Hamiltonians H_A and H_B. It is supposed that their eigenvalue problems are solved, and the complete set of the eigenvectors of the two independent systems are known.

In the processes investigated the Coulomb and the strong interactions play crucial role. The interaction Hamiltonian H_I comprises the Coulomb interaction potential V_{Cb} with the charged constituents of surroundings (solid) and the interaction potential V_{St} of the strong interaction:

$$H_I(\mathbf{x}_{AB}) = V_{Cb}(\mathbf{x}_{AB}) + V_{St}(\mathbf{x}_{AB}). \tag{10}$$

Therefore the charged particle assisted nuclear reactions are at least of second order in terms of standard perturbation calculation. The interaction $H_I(\mathbf{x}_{AB})$ between them is switched on adiabatically, the suffixes A and B in the argument symbolize that one party of the interaction comes from system A and the other from system B.

According to (10), the lowest order of S-matrix element of a charged particle assisted nuclear reaction has two terms. However, the contribution by the term, in which V_{St} according to chronological order precedes V_{Cb}, is negligible because of the smallness of the Coulomb factor which is appearing in the matrix element of V_{St} in this case (as it was discussed above). In the following we only deal with the dominant term. (In the nuclear matrix element the Coulomb interaction between the charged participants is taken into account using an appropriate approximate form (21) of (4).)

Thus in the process investigated, first a heavy, charged particle of system B takes part in a Coulomb scattering with any charged particle of system A and it is followed by a strong interaction $V_{St}(\mathbf{x}_{aB})$ with some nucleus of system A that leads to their final bound states. Particles 1, 3 belonging to system A are heavy constituents (e.g. atoms, ions [particle 1] and quasi-free deuterons [particle 3]) of the solid. Ingoing particles 2 belong to system B, that are charged, heavy particles (e.g. protons (p)), and it is supposed that they move freely at the surface of the solid (e.g. metal). The sub-lattice of the solid is filled partly with particles 3, which are deuterons further on, i.e. the number of filled lattice points of the d sub-lattice is $N_1\xi$, where N_1 is the number of possible lattice sites. It is supposed that only quasi-free deuterons take part in the process and they are produced by thermal activation. The number N_3 of quasi-free deuterons has strong temperature dependence.

$$N_3(T) = N_1 \xi e^{-\frac{U}{kT}} e^{-\frac{\varepsilon}{kT}}, \tag{11}$$

where U is the activation energy of the deuteron necessary for it to be quasi-free in the host lattice and ε is its quasi-kinetic energy. So $N_3(T)$ gives the number of those deuterons which are bound in the lattice but are delocalized, move quasi-free in it and take part in the process discussed.

Since the aim of this paper is to show the fundamentals of the main effect, the problem, that there may be identical, indistinguishable particles in systems A and B is not considered, the simplest description and the simplest process (the p capture reaction) is chosen.

For atomic potential we use screened Coulomb interaction potential of charge number z_1 with screening parameter $\lambda = z_1/a_0$ that results $V_{Cb}(\mathbf{r})$ of form

$$V_{Cb}(\mathbf{r}) = \frac{z_1 z_2}{2\pi^2} e^2 \int \frac{\exp(i\mathbf{q}\mathbf{r})}{q^2+\lambda^2} d\mathbf{q} \tag{12}$$

and coupling strength $e^2 = \alpha_f \hbar c$. $\hbar$ is the reduced Planck-constant, c is the velocity of light in vacuum and a_0 is the Bohr radius. Here z_2 is the charge number of the ingoing heavy charge particle (particle 2).

For the strong interaction the interaction potential

$$V_{St}(\mathbf{x}) = -V_0 \text{ if } |\mathbf{x}| \leq b \text{ and } V_{St}(\mathbf{x}) = 0 \text{ if } |\mathbf{x}| > b \tag{13}$$

is applied, where the choice for $V_0 = 25\ MeV$ and $b = 2\times10^{-13}\ cm$ seem to be reasonable in the case of deuteron like target particle [10].

According to standard perturbation theory of quantum mechanics the transition probability per unit time $\left(W_{fi}^{(2)}\right)$ of the process can be written as

$$W_{fi}^{(2)} = \frac{2\pi}{\hbar} \sum_f \left| T_{fi}^{(2)} \right|^2 \delta(E_f - \Delta) \tag{14}$$

with

$$T_{fi}^{(2)} = \sum_{2'} \frac{V_{St,f2'} V_{Cb,2'i}}{E_{2'} - E_i}, \tag{15}$$

where E_i, $E_{2'}$ and E_f are the kinetic energies in the initial, intermediate and final states, respectively, Δ is the reaction energy, i.e. the difference between the rest energies of the initial (E_{i0}) and final (E_{f0}) states ($\Delta = E_{i0} - E_{f0}$). $V_{Cb,2'i}$ is the matrix element of the Coulomb potential between the initial and intermediate states and $V_{St,f2'}$ is the matrix element of the potential of the strong interaction between intermediate and final states. Only capture processes are dealt with. The initial momenta and kinetic energies of particles 1, 2 and 3 are neglected ($E_i = 0$).

$$E_f = \frac{\hbar^2 \mathbf{k}_{1'}^2}{2m_1} + \frac{\hbar^2 \mathbf{k}_4^2}{2m_4}, \tag{16}$$

$$E_{2'} = \frac{\hbar^2 \mathbf{k}_{2'}^2}{2m_2} + \frac{\hbar^2 \mathbf{k}_{1'}^2}{2m_1}, \tag{17}$$

are the kinetic energies in the final and intermediate states. $\mathbf{k}_{1'}$, $\mathbf{k}_{2'}$ and $\mathbf{k}_4$ are wave vectors of particle 1', 2' and 4, respectively (see Fig. 1a).

V. CROSS SECTION OF RECOIL ASSISTED NUCLEAR REACTION IN d-FILLED METALS

The initial states of heavy constituents (e.g. atom or ion of the host lattice, particle 1 of rest mass m_1), that is localized around all of the lattice points, are described by a Bloch-function of the form [11]

$$\varphi_{\mathbf{k}_{1,i}}(\mathbf{r}_1) = \frac{1}{\sqrt{N_1}} \sum_{\mathbf{L}} e^{i\mathbf{k}_{1,i}\cdot\mathbf{L}} a(\mathbf{r}_1 - \mathbf{L}), \tag{18}$$

where $\mathbf{r}_1$ is the coordinate, $\mathbf{k}_{1,i}$ is wave vector of the first Brillouin zone (BZ) of the reciprocal lattice, $a(\mathbf{r}_1 - \mathbf{L})$ is Wannier-function, which is independent of $\mathbf{k}_{1,i}$ within the BZ and is well localized around lattice site $\mathbf{L}$. N_1 is the number of lattice points of the lattice of particles 1. (The

Coulomb-interaction between the ingoing free particles and the crystal may change the energy distribution of the states of form (18).)

$$a(\mathbf{x}) = \left(\frac{\beta^2}{\pi}\right)^{3/4} e^{-\frac{\beta^2}{2}\mathbf{x}^2}, \qquad (19)$$

that is the wave function of the ground state of a 3-dimensional harmonic oscillator of angular frequency ω with $\beta = \sqrt{m_1\omega/\hbar}$ [12].

For particles 2 (ingoing heavy particles, in our case protons) taking part in Coulomb interaction with the heavy constituent of the solid (with particle 1) we use plane waves before and after scattering as well. The final state of heavy constituent which comes from the solid (particle 1') is also a plane wave.

Using the states defined above for Coulomb matrix element we have

$$\begin{aligned} V_{Cb,2'i} &= \frac{z_1 z_2}{2\pi^2} e^2 \left(\frac{\beta^2}{\pi}\right)^{3/4} \frac{\delta(\mathbf{k}_{2'} + \mathbf{k}_{1'})}{\mathbf{k}_{1'}^2 + \lambda^2} \\ &\times \frac{(2\pi)^6}{V^{3/2}} \frac{1}{\sqrt{N_1}} \sum_{\mathbf{L}} \exp(i\mathbf{k}_{1,i}\mathbf{L}). \end{aligned} \qquad (20)$$

Here the approximation $\widetilde{a}(\mathbf{k}) = 8\pi^{9/4}\beta^{3/2}\delta(\mathbf{k})$ in the case of $\beta \ll k$ is used where $\widetilde{a}(\mathbf{k}) = \int e^{-i\mathbf{k}\cdot\mathbf{x}} a(\mathbf{x}) d^3x$ is the Fourier transform of (19) which is $\widetilde{a}(\mathbf{k}) = 2^{3/2}\pi^{3/4}\beta^{-3/2}\exp\left(-\frac{k^2}{2\beta^2}\right)$ that allows this approximation since $\beta \ll |\mathbf{k}_{1'}|$, $|\mathbf{k}_{2'}|$ and $|\mathbf{k}_4|$ as will be seen later. Furthermore $\mathbf{k}_{1,i}$ is neglected in the argument of the Dirac-delta since $|\mathbf{k}_{1,i}| \ll |\mathbf{k}_{1'}|$ (see later too).

Particle 3 is supposed to be free too therefore its state may also be considered to be a plane wave. When calculating the matrix element of the strong interaction potential between particles 2' and 3 we use an approximate form $\varphi_{2'3,Cb}(\mathbf{r})$ of the Coulomb solution

$$\varphi_{2'3,Cb}(\mathbf{r}) = e^{i\mathbf{k}_{2'}\cdot\mathbf{r}} f_{2'3}(k_{2'})/\sqrt{V}, \qquad (21)$$

namely a plane wave corrected by the appropriate Coulomb factor $f_{2'3}(k_{2'})$ corresponding to the particles 2' and 3, that take part in strong interaction. Here $\mathbf{r} = \mathbf{r}_{2'} - \mathbf{r}_3$ is their relative coordinate. (The approximation of the Coulomb solution of form (21) is valid in the range of the nucleus only.) The wave function of particles 2' and 3 can be written as the function of $\mathbf{r}$ and $\mathbf{R}$, which is the center of mass coordinate of particles $2'$ and 3, as

$$\varphi_{2'3}(\mathbf{R}, \mathbf{r}) = e^{i\mathbf{k}_{2'}\cdot(\mathbf{R}+\mathbf{r})} f_{2'3}(k_{2'})/V. \qquad (22)$$

The final state has the form

$$\varphi_4(\mathbf{R}, \mathbf{r}) = e^{i\mathbf{k}_4\cdot\mathbf{R}} \Phi_f(\mathbf{r})/\sqrt{V}, \qquad (23)$$

where $\Phi_f(\mathbf{r})$ is the nuclear state of the proton in particle 4. In the nuclear part of the model the Weisskopf approximation is used, i.e. for the final nuclear state of the proton we take

$$\Phi_{fW}(\mathbf{r}) = \sqrt{\frac{3}{4\pi R_0^3}} \qquad (24)$$

if $r \leq R_0$, where R_0 is the nuclear radius, and $\Phi_{fW}(\mathbf{r}) = 0$ for $r > R_0$. Since $\Phi_{fW}(\mathbf{r})$ and $V_{St}(\mathbf{r})$ both have spherical symmetry the spherical term $\sin(k_{2'}r)/(k_{2'}r)$ remains from $e^{i\mathbf{k}_{2'}\cdot\mathbf{r}}$ in the nuclear matrix-element. With the aid of the above wave function and $b = R_0$ assumption

$$V_{St,f2'}^W = -V_0 \frac{\sqrt{12\pi b}}{k_{2'}} f_{2'3}(k_{2'}) H(k_{2'}) \frac{(2\pi)^3}{V^{3/2}} \delta(\mathbf{k}_{2'} - \mathbf{k}_4) \qquad (25)$$

where

$$H(k_{2'}) = \int_0^1 \sin(k_{2'}bx) x dx. \qquad (26)$$

Collecting everything obtained above, applying correspondence (71) in (15) and integrating over $\mathbf{k}_{2'}$

$$\begin{aligned} T_{fi}^{(2)} &= -\frac{\left(3\beta^3 b\right)^{1/2}}{\pi^{9/4}} \frac{(2\pi)^6}{|\mathbf{k}_{1'}| V^2} \frac{\delta(\mathbf{k}_4 + \mathbf{k}_{1'})}{\left(|\mathbf{k}_{1'}|^2 + \lambda^2\right)} \\ &\times \frac{f_{2'3}(k_{1'}) H(k_{1'})}{\frac{\hbar^2 |\mathbf{k}_{1'}|^2}{2m_0 a_{12}}} \frac{z_1 z_2 e^2 V_0}{\sqrt{N_1}} \sum_{\mathbf{L}} \exp(i\mathbf{k}_{1,i}\mathbf{L}). \end{aligned} \qquad (27)$$

Substituting (27) into (14), the following correspondence and identities are used: (72) − (75). Carrying out integration over $\mathbf{k}_{1'}$ with the aid of $\delta(\mathbf{k}_4 + \mathbf{k}_{1'})$, applying the relations (76) and (77), neglecting λ since $\lambda \ll |k_0|$, taking $z_2 = 1$, $A_2 = 1$ (particle 2 is a proton), integrating over $\mathbf{k}_4$ and dividing by v/V which is the flux of ingoing particles 2 of velocity v one can obtain the cross section σ_{RANR} of the process as

$$\sigma_{RANR} = \frac{S}{\sqrt{E_{2i}}} \qquad (28)$$

where

$$S = 24\sqrt{\pi}\beta^3 b \frac{\left(z_1 z_2 \alpha_f V_0\right)^2 (\hbar c)^4}{\Delta^{9/2} (m_0 c^2)} \frac{a_{12}^2}{a_{14}^{7/2}} f_{2'3}^2(\Delta) H^2(k_0). \qquad (29)$$

(The subscript $RANR$ refers to the abreviation of recoil assisted nuclear reaction.) Now it may be checked that the supposed $\beta \ll |\mathbf{k}_{1'}|$, $|\mathbf{k}_{2'}|$, $|\mathbf{k}_4|$ and $|\mathbf{k}_{1,i}| \ll |\mathbf{k}_{1'}|$ conditions are fulfilled since $|\mathbf{k}_{1'}| = |\mathbf{k}_{2'}| = |\mathbf{k}_4| = k_0$, ($k_0 = \hbar^{-1}\sqrt{2m_0 a_{14}\Delta}$).

In a deuterized Pd the ground state of the oscillator which describes the vibration of particles 1 is $E_0 = 72$ meV leading to $\hbar\omega = 48$ meV [13] and $\beta = 3.4 \times 10^8 \times A_1^{1/2}$ cm^{-1}. Applying this and taking $A_1 = 106$ as a characteristic Pd atomic number, $b\beta^3 = 8.58 \times 10^{15}$ cm^{-2}. In the case of Pd assisted $p + d \rightarrow {}^3He$ reaction $z_1 = 46$ and the $a_{12} = 1$ and $a_{14} = 3$ approximations may be applied. Furthermore, with $\Delta = 5.493\ MeV$, $H(k_0 b) = 0.43$ and $f_{2'3}^2 = 0.88$ the cross-section results $\sigma_{LENR} = 6.67 \times E_{2i}^{-1/2}\ \mu b$ (i.e. $S = 6.67\ eV^{1/2}\mu b$, and E_{2i} is measured in eV).

Based on the above results one can estimate the energy range of astrophysical factor measurements where recoil assisted nuclear reactions have to be taken into account. It is determined by the following relation $\sigma_{RANR} \exp[-U/(kT)] \gtrsim \sigma(E)$ where $\sigma(E)$ is determined by (8). This relation can be converted into the condition

$$\ln\left(\frac{S}{S(0)}\sqrt{E_{2i,lab}}\right) - \frac{U}{kT} \gtrsim -2\pi\alpha_f\sqrt{\frac{m_0 c^2}{E_{2i,lab}}} \quad (30)$$

which is valid in the laboratory frame of reference. Here $E_{2i,lab}$ is the initial energy of particle 2, i.e. the beam energy, and it is also supposed that the S value of the recoil assisted dd reactions may be estimated by the S value obtained above. If one takes $U = 0.23\ eV$ [14] and $kT = 0.025\ eV$ which corresponds to room temperature then $U/(kT) = 9.2$. Moreover, $S(0) = 5.5 \times 10^4\ eVb$ and $5.6 \times 10^4\ eVb$ of the $d + d \rightarrow {}^3He + n$ and $d + d \rightarrow p + t$ reactions [5]. Applying these numbers in (30) one obtains that near and below $E_{2i,lab} = 8\ keV$ the recoil assisted reactions have to be taken into account when evaluating astrophysical measurements of dd reactions. This result fits in with observations [6].

VI. LOW ENERGY NUCLEAR REACTIONS ALLOWED BY RECOIL ASSISTANCE - POSSIBILITY OF NUCLEAR TRANSMUTATIONS

In order to get more information on the capability of the recoil mechanism discussed, some cases of the recoil assisted proton captures [see (1)] are investigated. Particle 1 is a heavy particle, which assists the nuclear reaction and particle 2 is proton. Particles 3 and 4 have mass and charge numbers A_3, z_3 and $A_3 + 1$, $z_3 + 1$ respectively (see Fig. 1). The reaction energy Δ may be calculated using mass excess data [9]. Δ is the difference between the sum of the initial and final mass excesses, i.e. $\Delta = \Delta_p + \Delta_{A_3,z_3} - \Delta_{A_3+1,z_3+1}$, where Δ_p, Δ_{A_3,z_3} and Δ_{A_3+1,z_3+1} are mass excesses of proton, ${}^{A_3}_{z_3}X$ and ${}^{A_3+1}_{z_3+1}Y$, respectively. Investigating the mass excess data [9] one can recognize that the process is energetically allowed in the case of great number of nuclei and, consequently, may lead to nuclear transmutations [3].

Considering experiments using Pd cathode during electrolysis an important family of the recoil assisted proton capture reads as

$${}^{A_1}_{46}Pd + p + {}^{A_3}_{46}Pd \rightarrow {}^{A_1}_{46}Pd' + {}^{A_3+1}_{47}Ag + \Delta, \quad (31)$$

where the recoil assisted proton capture is mainly followed by the

$${}^{A+1}_{47}Ag + e \rightarrow {}^{A+1}_{46}Pd + Q_{EC} \quad (32)$$

A_3	102	104	105	106	108	110
Δ(MeV)	4.155	4.966	5.814	5.789	6.487	7.156
r_{A_3}	0.0102	0.1114	0.2233	0.2733	0.2646	0.1172

TABLE I: Numerical data of the ${}^{A_1}_{46}Pd + p + {}^{A_3}_{46}Pd \rightarrow {}^{A_1}_{46}Pd' + {}^{A_3+1}_{47}Ag + \Delta$ reactions. r_A is the relative natural abundance. Δ is the energy of the reaction.

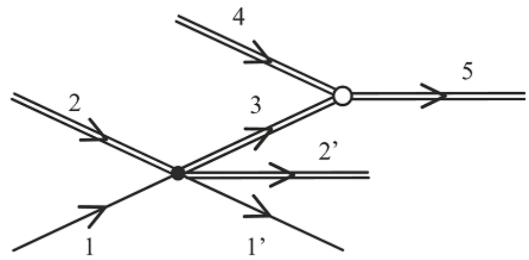


FIG. 2: The graph of cooperative exchange process by heavy particle (e.g. neutro, proton, etc.) exchange. Particle 1 and 1' are any charged particle which assist the reaction, particle 2 is the nucleus which looses the heavy particle (e.g. neutron, proton, etc.) and becomes particle 2'. Particle 3 is the intermediate heavy particle (e.g. neutron, proton, etc.). Particle 4 is the nucleus which absorbs the heavy particle (e.g. neutron, proton, etc.) and becomes particle 5. The filled dot denotes (in the case of neutron the dipole term of) the Coulomb-interaction and the open circle denotes nuclear (strong) interaction.

reaction. (The relevant data can be found in TABLE I., the estimated $f_{2'3}^2 = 0.196$, which may be attached to the reactions, is determined in Section III., the nuclear data of TABLE I. are taken from [9].)

The processes (31) may produce stable ${}^{107}_{47}Ag$ and ${}^{109}_{47}Ag$ isotopes whose recoil assisted proton capture reaction may give rise to a chain of nuclear transmutations.

Moreover, the capture reaction may be extended to the recoil assisted capture of heavy particles ${}^{A_2}_{z_2}w$ (see reaction (3)), e.g. the capture of deuteron (d), triton (t), ${}^3He$, ${}^4He$, etc.. The number of energetically allowed reactions of this type is very large too, their perceptibility is limited by the actual value of the factor $f_{2'3}^2$. The reactions of type (3) are also the sources of nuclear transmutations [3].

Another family of recoil assisted reactions, which is worth mentioning, is the family of exchange reactions. In recent works [15] the cooperative internal conversion processes by neutron and proton exchange were investigated. The cooperative exchange processes may be generally written in the form

$$q + {}^{A_2}_{z_2}X + {}^{A_4}_{z_4}Y \rightarrow q' + {}^{A_2-A_3}_{z_2-z_3}V + {}^{A_4+A_3}_{z_4+z_3}W + \Delta. \quad (33)$$

Here q stands for any charged particle (initially free or bound electron and any nucleus of charge number z_1)

which assists the reaction in which a heavy particle ${}^{A_3}_{z_3}w$ (n, p, d, t, ${}^3He$, ${}^4He$, etc.) is exchanged between the initial nuclei ${}^{A_2}_{z_2}X$ and ${}^{A_4}_{z_4}Y$ resulting the final particles ${}^{A_2-A_3}_{z_2-z_3}V$ and ${}^{A_4+A_3}_{z_4+z_3}W$ (see Fig. 2.). These reactions lead to nuclear transmutations too [3] and the number of energetically allowed reactions of this type is also very large. In exchange reactions the reaction energy Δ is divided between particles q', ${}^{A_2-A_3}_{z_2-z_3}V$ and ${}^{A_4+A_3}_{z_4+z_3}W$. It was shown in [15] that in the case of electron assistance the electrons take off minor parts of Δ.

VII. CRITICAL ANALYSIS OF FLEISCHMANN-PONS TYPE EXPERIMENTS

In the experiment of [1] Pd was filled with deuterons during electrolysis. The electrolyte had $LiOD$ content too. The most straightforward Pd assisted d capture reaction is

$$ {}^{A}_{46}Pd + d + d \rightarrow {}^{A}_{46}Pd' + {}^{4}_{2}He + \Delta. \quad (34)$$

In this reaction $\Delta = 23.847$ MeV and it is carried by the particles ${}^{A}_{46}Pd'$ and ${}^{4}_{2}He$ which have momentum of equal magnitude but opposite direction. Since both particles are charged and heavy they loose their kinetic energy in a very short range and so they convert the reaction energy into heat.

However, it is also possible that the two deuterons take part in the

$$ {}^{A}_{46}Pd + d + d \rightarrow {}^{A}_{46}Pd' + n + {}^{3}_{2}He + 3.269\ MeV \quad (35)$$

and

$$ {}^{A}_{46}Pd + d + d \rightarrow {}^{A}_{46}Pd' + p + t + 4.033\ MeV \quad (36)$$

reactions. Obviously any other charged particle q of the environment may assist the above three reactions. The following qualitative consideration suggests that the rate of (34) is expected to be larger than that of (35) and (36). In the case of (34) $\mathbf{p}_4 = \mathbf{p}_{2'} (= -\mathbf{p}_{1'})$ and in the case of (35) and (36) $\mathbf{p}_5 + \mathbf{p}_6 = \mathbf{p}_{2'} (= -\mathbf{p}_{1'})$. ($\mathbf{p}_{1'}$, $\mathbf{p}_4$, $\mathbf{p}_5$ and $\mathbf{p}_6$ are the momenta of particles 1', 2', 4, 5 and 6, see Fig. 1.) Thus in the later case the effective momentum transfer to particle 2' is limited if $\mathbf{p}_5 - \mathbf{p}_6$ is large and this limit decreases the effective phase space where large momentum transfer to particle 2' happens. Large momentum transfer is necessary to the large value of $f^2_{2'3}$.

A. Rate and power of sample in electrolysis

The above model (see Sections IV. and V.) of recoil assisted nuclear reactions in d-filled metals will be applyed. Although this is a very rough model of electrolysis it shows some parameters which are crucial from the point of view of the phenomenon. Using the cross-section result obtained ((28) and (29)) and estimating S with the numerical value of S of the recoil assisted $p+d \rightarrow {}^3He$ reaction we can estimate the rate and the generated power during electrolysis. In this naive model it is supposed that reaction (34) is the only one which assumption is not correct as will be seen later.

Supposing surface effect the rate of the process can be written as

$$\frac{dN}{dt} = N_1 N_3 \sigma_{RANR} \Phi, \quad (37)$$

where Φ is the flux of particles 2 (Φ is now the deuteron flux).

In a numerical example it is supposed that deuterons are very slow, e.g. of energy $E_{1i} = 1$ eV. The deuteron current is $I_p = \eta I$, where I is the total current, and the surface of the electrode is F (in cm^2) resulting $\Phi = \eta I/(eF) = \eta j/e$ where $e = 1.602 \times 10^{-19}$ As is the elementary charge and j is the current density in Acm^{-2}. The parameter η may be connected to the pH value of the electrolyte. The very slow deuterons may interact only with first Pd atomic plane of volume Fd which contains $N_1 = Fd/v_{cell} = 4F/d^2$ number of lattice points of the Pd lattice. v_{cell} is the volume of the elementary cell of the solid ($v_{cell} = d^3/4 = 1.47 \times 10^{-23}$ cm^3 with $d = 3.89 \times 10^{-8}$ cm which is the lattice parameter of the Pd lattice).

Using (11) the rate of the process in the sample can be estimated as

$$\frac{dN}{dt} \simeq \frac{16jF^2}{ed^4}\xi\eta\sigma_{RANR}e^{-\frac{U}{kT}} = 2.9 \times 10^{20} \times jF^2\xi\eta e^{-\frac{U}{kT}}. \quad (38)$$

Taking $U = 0.23$ eV [14] and $kT = 0.025$ eV (room temperature) $e^{-\frac{U}{kT}} = 1.01 \times 10^{-4}$ which results $dN/dt = 2.9 \times 10^{16} \times jF^2\xi\eta$. In this case the power $P = \Delta\,(dN/dt) = 1.1 \times 10^5 \times jF^2\xi\eta$ in W, which results $P = 1$ W at $\xi = 1$, $F = 1$ cm^2, $j = 1$ Acm^{-2} and with $\eta = 9.1 \times 10^{-6}$.

As a conclusion, the current density j of the electrolysis, the accountable surface F of the Pd cathode, the deuteron concentration ξ in Pd, the pH value (connected to η) the Li concentration and the temperature T both at the surface of the Pd cathode are crucial quantities from the point of view of the created power by recoil assisted nuclear reaction at the surface. Unfortunately, the published observations are generally incomplete, the pH value is usually missing which may be the reason that the generated power reported seems to be sometimes on and sometimes off. The different kinds (e.g. smooth or cauliflower like) of the surface of the cathode may be the reason that the result of some seemingly identical experiments differ since it is very hard to estimate the effective surface of e.g. a cauliflower like surface.

Naturally the recoil assisted $p+d \rightarrow {}^3_2He$ reaction may also take place if the there is some normal H concentration present. It may be desirable for efficient energy production (see below).

A	102	104	105	106	108	110
Δ_+	-0.446	-0.978	1.491	-1.533	-1.918	-2.320
r_A	0.0102	0.1114	0.2233	0.2733	0.2646	0.1172

TABLE II: Numerical data of reactions (42) and (43). The reaction is energetically allowed if $\Delta = \Delta_-(d) + \Delta_+(A) > 0$ (in the case of (42)) and $\Delta = \Delta_-(Li) + \Delta_+(A) > 0$ (in the case of (43). A is the mass number, r_A is the relative natural abundance, For $\Delta_-(d) = 5.847$ MeV and $\Delta_-(Li) = 0.821$ MeV. $\Delta_+(A) = \Delta_A - \Delta_{A+1}$ are given in MeV units. Δ_A and Δ_{A+1} are the mass excess data of isotopes of mass numbers A and $A+1$

B. Other recoil assisted nuclear reactions in experiments of Fleischmann-Pons type

Moreover, charged particle (q) assisted d capture reactions may take place with ${}^6_3Li$ and ${}^7_3Li$ isotopes

$$q + d + {}^6_3Li \rightarrow q' + 2{}^4_2He + 22.372\ MeV \tag{39}$$

and

$$q + d + {}^7_3Li \rightarrow q' + 2{}^4_2He + n + 15.122\ MeV. \tag{40}$$

If the there is some normal H concentration present then the

$$q + p + {}^7_3Li \rightarrow q' + 2{}^4_2He + 17.347\ MeV. \tag{41}$$

reaction is also possible.

Now we deal with neutron exchange reactions (see (33) and Fig. 2). Two types of charged particle (q) assisted neutron exchange processes with Pd nuclei are also possible:

$$q + d +{}^A_{46} Pd \rightarrow q' + p +{}^{A+1}_{46} Pd + \Delta \tag{42}$$

with $\Delta = \Delta_-(d) + \Delta_+(A)$ and

$$q +{}^7_3 Li +{}^A_{46} Pd \rightarrow q' +{}^6_3 Li +{}^{A+1}_{46} Pd + \Delta \tag{43}$$

with $\Delta = \Delta_-(Li) + \Delta_+(A)$ (the $\Delta_+(A)$ values can be found in Table II). $\Delta_-(d) = \Delta_d - \Delta_p = 5.847$ MeV and $\Delta_-(Li) = \Delta({}^7_3Li) - \Delta({}^6_3Li) = 0.821$ MeV are the energies of neutron loss of d and ${}^7_3Li$, where Δ_d, Δ_p, $\Delta({}^7_3Li)$ and $\Delta({}^6_3Li)$ are the mass excesses of deuteron, proton, ${}^7_3Li$ and ${}^6_3Li$, respectively. In reactions (42) and (43) q denotes any charged particle (e.g. electron of the metal, Pd) and it is particle 1, d and ${}^7_3Li$ are particle 2, respectively, and ${}^A_{46}Pd$ appears as particle 4 (see Fig. 2). Reaction (42) is energetically allowed for all the natural isotopes of Pd since $\Delta = \Delta_-(d) + \Delta_+(A) > 0$ for each A (see the $\Delta_+(A)$ values of Table II). In the case of reaction (43) the $\Delta = \Delta_-(Li) + \Delta_+(A) > 0$ condition holds at $A = 102$ and $A = 105$ resulting $\Delta = 0.375$ MeV and $\Delta = 2.312$ MeV, respectively.

However, at the Pd surface other types of charged particle assisted neutron and proton exchange processes (see (33) and Fig. 2) with d and Li nuclei of the electrolyte and d solved in Pd are possible:

$$q + d + d \rightarrow q' + p + t + 4.033\ MeV, \tag{44}$$

$$q + d + d \rightarrow q' + n + {}^3_2He + 3.269\ MeV, \tag{45}$$

$$q + d + {}^6_3Li \rightarrow q' + p + {}^7_3Li + 5.025\ MeV, \tag{46}$$

$$q + d + {}^7_3Li \rightarrow q' + 2{}^4_2He + n + 15.122\ MeV \tag{47}$$

and

$$q + d + {}^7_3Li \rightarrow q' + {}^8_4Be + n + 15.030\ MeV, \tag{48}$$

which is promptly followed by the decay ${}^8_4Be \rightarrow 2{}^4_2He$ (with $\Gamma_\alpha = 6.8$ eV). In reactions (44)-(48) d is particle 2 (see Fig. 2). (The list of reactions is incomplete.)

C. Possibility of usual nuclear rections

In reaction (42) protons of energy up to 7.269 MeV and in reaction (43) ${}^6_3Li$ particles of maximum energy 2.189 MeV are created. These energetic particles may enter into usual nuclear reactions with the nuclei of deuteron loaded Pd and electrolyte. The reactions are (without completeness):

$$p + d \rightarrow^3_2 He + \gamma \text{ with } Q = \Delta + E_{kin}(p), \tag{49}$$

$$p +^7_3 Li \rightarrow 2{}^4_2He + Q \text{ with } Q = \Delta + E_{kin}(p), \tag{50}$$

$${}^6_3Li + d \rightarrow 2{}^4_2He + Q \text{ with } Q = \Delta + E_{kin}(Li), \tag{51}$$

$${}^6_3Li + d \rightarrow p +^7_3 Li + Q \text{ with } Q = \Delta + E_{kin}(Li). \tag{52}$$

In (50) and (51) the emitted ${}^4_2He$ has energy $E_{^4He} > 8.674$ MeV and $E_{^4He} > 11.186$ MeV, and in (52) the created p and ${}^7_3Li$ have energy $E_p > 4.397$ MeV and $E_{^7Li} > 0.628$ MeV, respectively. It can be seen that in (50) and (51) ${}^4_2He$ is produced. The ${}^7_3Li$ particles may enter into reaction

$${}^7_3Li + d \rightarrow 2{}^4_2He + n + Q \text{ with } Q = \Delta + E_{kin}(Li) \tag{53}$$

that contributes to ${}^4_2He$ production too. Here and above $E_{kin}(p)$ and $E_{kin}(Li)$ are the kinetic energies of the initial protons, ${}^6_3Li$ and ${}^7_3Li$ isotopes. In reactions $(42) - (53)$ heavy, charged particles of kinetic energy lying in the MeV range are created which are able to assist nuclear reactions too.

The energetic ${}^3_2He$ and ${}^4_2He$ particles may enter in the following reactions:

$${}^3_2He + d \rightarrow {}^4_2He + p + 18.353\ MeV + E_{kin}({}^3_2He), \tag{54}$$

$$ {}^3_2He + t \to {}^6_3Li + 15.795\ MeV + E_{kin}({}^3_2He), \quad (55) $$

$$ {}^4_2He + d \to {}^6_3Li + 1.743\ MeV + E_{kin}({}^4_2He), \quad (56) $$

$$ {}^4_2He + t \to {}^7_3Li + 2.467\ MeV + E_{kin}({}^4_2He), \quad (57) $$

$$ {}^3_2He + {}^4_2He \to {}^7_4Be + 1.587\ MeV + E_{kin}({}^3_2He + {}^4_2He), \quad (58) $$

which decays with $\tau = 53.29\ d$ into ${}^7_3Li$ by electron capture (EC) and

$$ {}^3_2He + {}^3_2He \to {}^6_4Be + 11.488\ MeV + E_{kin}({}^3_2He + {}^3_2He) \quad (59) $$

which decays with $\Gamma = 92\ keV$ into $2p + \alpha$.

Moreover the following normal reactions may also take place

$$ {}^3_2He + {}^7_3Li \to {}^{10}_5B + 17.788\ MeV + E_{kin}({}^3_2He), \quad (60) $$

$$ {}^4_2He + {}^6_3Li \to {}^{10}_5B + 4.461\ MeV + E_{kin}({}^4_2He), \quad (61) $$

and

$$ {}^4_2He + {}^7_3Li \to {}^{11}_5B + 8.665\ MeV + E_{kin}({}^4_2He). \quad (62) $$

The nuclei ${}^{10}_5B$ and ${}^{11}_5B$ are stable. Naturally, if the energy of initial particles is moderated to low value in the apparatus then all the above reactions can take place in recoil assited channels.

From the above one can see that many types of reactions exist which are capable of energy production and in many of them energy production is accompanied by ${}^4_2He$ production. In the majority of the above reactions heavy charged particles are created with short range and so they loose their energy in the material of the experimental apparatus mainly in the electrode (cathode) and the electrolyte, therefore their direct observation is difficult. It is mainly heat production, which is a consequence of deceleration in the material of the apparatus, that can be experienced. About third of the processes, mainly the secondary processes are the sources of neutron emission. X- and $\gamma-$rays may be originated mainly from bremsstrahlung. The above reasoning tallies with experimental observations [3].

One can obtain the possible recoil assisted reactions if in the reactions q is replaced by heavy charged particles (p, t, ${}^3_2He$, ${}^4_2He$, ${}^6_3Li$, ${}^7_3Li$, ${}^8_4Be$ and ${}^{A+1}_{46}Pd$ with $A = 102, 104 - 106, 108, 110$) some of them are created also in normal reactions. Consequently, it is rather a great theoretical challenge and task to determine precisely the relative rates and their couplings of all the accountable reactions, a work which is, nevertheless, necessary for accurate quantitative analysis of experiments.

The relative rates of coupled reactions of many types depend significantly on the geometry, the kind of matter and other parameters of the experimental apparatus and on some further variables, which may be attached to a concrete experiment. This situation may also be responsible for the diversity of the results of experiments, which are thought to have been carried out in seemingly similar circumstances.

D. Further nuclear transmutations

If there are deuterons present in the experiment then the most plausible process is the

$$ q + d + {}^{A_3}_{z_3}X \to q' + {}^{A_3+2}_{z_3+1}Y + \Delta \quad (63) $$

recoil assisted d capture process. This type of reaction may cause nuclear transmutations explained by supposing successive d capture reported recently [16].

If Li is present then the

$$ q + {}^{A_2}_3Li + {}^{A_3}_{z_3}X \to q' + {}^{A_2+A_3}_{z_3+3}Y + \Delta \quad (64) $$

recoil assisted Li capture reactions may also lead to nuclear transmutations (and energy production). As an example let us see the Pd assisted

$$ {}^{A_1}_{46}Pd + {}^{A_2}_3Li + {}^{A_3}_3Li \to {}^{A_1}_{46}Pd' + {}^{A_2+A_3}_6C + \Delta \quad (65) $$

Li capture reactions. Using (9) with $z_2 = z_3 = 3$, $A_2 = 7$, $A_2 + A_3 = A_4 = 13$ and $A_1 = 106$, which corresponds to the

$$ {}^{106}_{46}Pd + {}^7_3Li + {}^6_3Li \to {}^{106}_{46}Pd' + {}^{13}_6C + 25.869\ MeV \quad (66) $$

reaction, $\eta_{2'3} = 0.573$ and $f^2_{2'3} = 0.101$.

The reaction

$$ {}^{A_1}_{46}Pd + {}^6_3Li + {}^6_3Li \to {}^{A_1}_{46}Pd' + 3{}^4_2He + 20.898\ MeV \quad (67) $$

may have importance too.

It seems to be surprising, but the Pd assisted

$$ {}^{A_1}_{46}Pd + {}^{A_2}_3Li + {}^{A_3}_{46}Pd \to {}^{A}_{46}Pd' + {}^{A_2+A_3}_{49}In + \Delta \quad (68) $$

recoil assisted Li capture reactions in Pd has small, but not negligible rate. As a numerical example let us take $z_2 = 3$, $z_3 = 46$, $A_2 = 7$, $A_3 = 106$, $A_4 = 113$ and $A_1 = 106$ in (9) resulting $\eta_{2'3} = 5.42$ and $f^2_{2'3} = 5.3 \times 10^{-14}$ with $\Delta = 14.369\ MeV$, which is the reaction energy of the ${}^{106}_{46}Pd + {}^7_3Li + {}^{106}_{46}Pd \to {}^{106}_{46}Pd' + {}^{113}_{49}In$ recoil assisted Li capture reaction. In reactions (68) stable and unstable In isotopes may appear with the later decaying into Cd and Sn isotopes by electron capture and β^- emission.

Next let us see the possibility of normal reactions. For instance in a Fleischmann-type experiment ${}^6_3Li$ particles of energy up to 2.189 MeV are created in reaction (43) so the reaction

$$ {}^6_3Li + {}^6_3Li \to {}^{12}_6C + \gamma \text{ with } Q \quad (69) $$

may have minor, but measurable probability. Here $Q = 28.17\ MeV + E_{kin}(Li)$.

The Coulomb factor of reaction (69) is $f^2_{Li,Li} = 1.71 \times 10^{-3}$ at 2.189 MeV kinetic energy of ${}^6_3Li$ particles. The magnitude of the factor $f^2_{Li,Li}$ indicates that the rate of reaction (69) may be large enough to be able to produce carbon traces in observable quantity too.

Moreover, in reactions (42) and (43) free ${}^{A}_{46}Pd$ particles are created offering e.g. the possibility of the

$$q +^{A_2}_{46} Pd +^{A_4}_{46} Pd \rightarrow q' +^{A_2-3}_{44} Ru +^{A_4+3}_{48} Cd + \Delta \quad (70)$$

charged particle assisted ${}^{3}_{2}He$ exchange process (see (33) and Fig. 2). The particle q and the other Pd particle are in the solid. Analyzing mass excess data [9] it was found that e.g. the $q+^{103}_{46}Pd+^{111}_{46}Pd \rightarrow q'+^{100}_{44}Ru+^{114}_{48}Cd+\Delta$ (${}^{3}_{2}He$ exchange process) has reaction energy $\Delta = 5.7305$ MeV. [${}^{103}_{46}Pd$ and ${}^{111}_{46}Pd$ are produced in reaction (42).] Calculating the $f^2_{2'3} = f^2_{34}$ factors taking $A_2 = A_4 = 100$, $z_2 = z_4 = 46$, $A_3 = 3$, $z_3 = 2$ (see (33) and Fig. 2) in the calculation one gets $f^2_{2'3}f^2_{34} = 2.5 \times 10^{-12}$ which seems to be large enough to produce Cd and Ru traces in an experiment of time of many days long.

The above reactions are only examples and may offer starting point to the full explanation of nuclear transmutations arising in the experiment [3].

VIII. SUMMARY

It is found that, contrary to the commonly accepted opinion, nuclear reactions can happen between heavy, charged particles of like (positive) charge of low initial energy due to the assistance of other charged particles present. It is recognized, that one of the participant particles of a nuclear reaction of low initial energy may pick up great momentum in a Coulomb scattering process on a third particle of the surroundings. The virtually acquired great momentum, that is determined by the energy of the reaction, can help to overcome the hindering Coulomb barrier and can highly increase the rate of the nuclear reaction even in cases when the rate would be otherwise negligible. It is found that the recoil assisted $p+d \rightarrow {}^{3}He$ process may have a cross section of accountable magnitude at low energies.

There may be many fields of physics where the traces of the proposed mechanism may have been previously appeared. It is not the aim of this work to give a systematic overview of these fields. Here we only dealt with two of them that are thought to be partly related or explained by the processes proposed. The first is the so called anomalous screening effect observed in low energy accelerator physics investigating astrophysical factors of nuclear reactions of low atomic numbers [6]. The other one is the family of low energy nuclear fusion processes. The physical background, discussed in the Introduction and in Section II., was questioned by the two decade old announcement [1] on excess heat generation at deuterized Pd cathodes during electrolysis at near room temperature. The mechanisms discussed here can explain the main problems raised in connection of the phenomenon (listed as points $(a) - (d)$ in the introduction). (a) The mechanisms proposed here make low energy fusion reactions and (c) nuclear transmutations possible. (b) The processes discussed explain the lack of reaction products normally expected . (d) The uncertain reproduction is also made clear.

IX. APPENDIX

In the calculation the following correspondences and relations applied:

$$\sum_{2'} \rightarrow \int \left[V/\left(2\pi\right)^3 \right] d\mathbf{k}_{2'}, \quad (71)$$

$$\sum_{f} \rightarrow \int \left[V/\left(2\pi\right)^3 \right] d\mathbf{k}_{4} \times \int \left[V/\left(2\pi\right)^3 \right] d\mathbf{k}_{1'}, \quad (72)$$

$$\frac{1}{N_1} \sum_{\mathbf{L},\mathbf{L}'} \frac{1}{N_1} \sum_{\mathbf{k}_{1,i} \in BZ} \exp i\mathbf{k}_{1,i} \left(\mathbf{L} - \mathbf{L}' \right) = 1, \quad (73)$$

$$\left[\delta \left(\mathbf{k}_4 + \mathbf{k}_{1'} \right) \right]^2 = \delta \left(\mathbf{k}_4 + \mathbf{k}_{1'} \right) \delta \left(\mathbf{0} \right), \quad (74)$$

$$\delta \left(\mathbf{0} \right) = \left(2\pi \right)^{-3} V, \quad (75)$$

$$\delta(E_f - \Delta) = \delta \left(k_4 - k_0 \right) m_0 a_{14} \hbar^{-2} k_0^{-1} \quad (76)$$

with

$$k_0 = \hbar^{-1} \sqrt{2 m_0 a_{14} \Delta}. \quad (77)$$

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