

Effect of impurities on nuclear fusion

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Modification of nuclear reactions due to impurities in plasma is investigated. The hindering effect of Coulomb repulsion between reacting particles, that is effective in direct reactions, can practically disappear if Coulomb interaction of the reacting particles with impurities embedded in plasma is taken into account. The change of the wavefunction of reacting particles in nuclear range due to their Coulomb interaction with impurity is determined using standard time independent perturbation calculation of quantum mechanics. The result can be interpreted as if a slow, quasi-free particle (e.g. a proton) were pushed by a heavy, assisting particle (impurity) of the surroundings and can get (virtually) such a great magnitude of momentum which significantly increases the nuclear contact probability density and also the probability of its capture by another nucleus. As a sample reaction the process, called impurity assisted nuclear pd reaction is investigated and the rate and power densities produced by the reaction are numerically calculated. With the aid of astrophysical factors the rate and power densities of the impurity assisted $d(d, n)^3_2\text{He}$, $d(d, p)t$, $d(t, n)^4_2\text{He}$, $^3_2\text{He}(d, p)^4_2\text{He}$, $^6_3\text{Li}(p, \alpha)^3_2\text{He}$, $^7_3\text{Li}(p, \alpha)^4_2\text{He}$, $^9_4\text{Be}(p, \alpha)^6_3\text{Li}$, $^9_4\text{Be}(p, d)^8_4\text{Be}$, $^9_4\text{Be}(\alpha, n)^{12}_6\text{C}$, $^{10}_5\text{B}(p, \alpha)^7_4\text{Be}$ and $^{11}_5\text{B}(p, \alpha)^8_4\text{Be}$ reactions are also estimated. The affect of plasma-wall interaction on the process is also considered. A partial survey of impurity assisted nuclear reactions which may have practical importance in energy production is also presented.

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I. INTRODUCTION

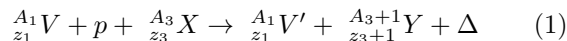
Nuclear fusion reactors need to be heated to very high temperature to overcome the Coulomb repulsion between nuclei to fuse. [1]. (Mathematically it is manifested in the exponential energy dependent factor in the cross section of fusion reactions [see (6)]. For details see Section II.)

The effect of surroundings on nuclear fusion rate in astrophysical condensed and dense laboratory plasmas has been extensively studied in the case of usual nuclear reactions. In tenuous plasmas the effect of spectator nuclei and electrons (the environment) on the Gamow-rate of reacting nuclei which are assumed to interact with bare Coulomb potential is negligible [2]. Moreover in some (e.g. tokamak-like) devices the presence of impurities during the heat up and working periods is undesirable because of high loss power generated by them [3].

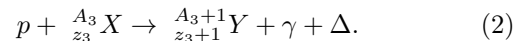
In this paper it will be shown however, that spectator nuclei can significantly enhance nuclear processes and allow new types of reactions.

We are going to investigate these processes of new type that can take place due to impurities and their effect on nuclear fusion devices. We focus our attention on Coulomb interaction between fuel nuclei and environment, namely on consequences of interactions with impurities which can activate new type of reactions of cross section of considerable magnitude which may change the condition of necessary plasma temperature and what is more remarkable, the mechanism found does not need plasma state at all.

We investigate the

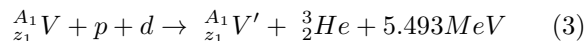


process called impurity assisted proton capture, a process among atoms or atomic ions containing ${}_{z_1}^{A_1}V$ nuclei (e.g. Xe) and protons or hydrogen atoms and ions or atoms of nuclei ${}_{z_3}^{A_3}X$ (e.g. deuterons) that are initially supposed to be in a plasma. Δ is the energy of the reaction. In normal case proton capture may happen in the



reaction where γ emission is required by energy and momentum conservation. Accordingly (1) describes a new type of p -capture. In the usual p -capture reaction (2) particles ${}_{z_3+1}^{A_3+1}Y$ and γ take away the reaction energy and the reaction is governed by electromagnetic interaction. In reaction (1) the reaction energy is taken away by particles ${}_{z_1}^{A_1}V'$ and ${}_{z_3+1}^{A_3+1}Y$ while the reaction is governed by Coulomb as well as strong interactions.

First we pay our attention to the impurity assisted $p + d \rightarrow {}^3\text{He}$ reaction



in an impurity contaminated plasma, that will be discussed in more detail.

Cross section and rate of process (1) to be considered can be calculated by the rules of time-independent perturbation calculation of quantum mechanics [4]. Our results indicate that the cross section of process (1), which is an indirect (second order) reaction, may be essentially

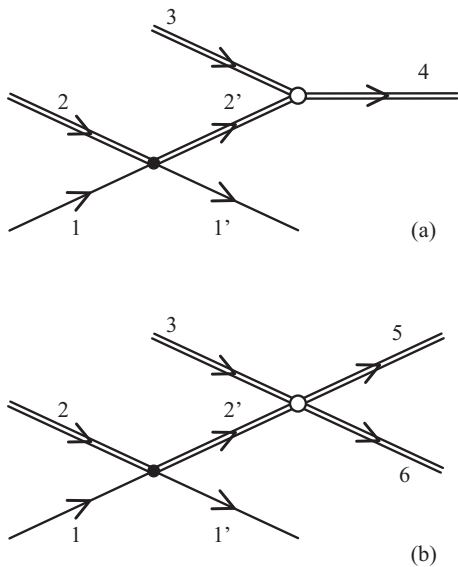


FIG. 1: The graphs of impurity assisted nuclear reactions. The single lines represent (initial (1) and final (1')) impurity particle of the plasma. The double lines represent free, heavy initial (2) particles (such as p , d), their intermediate state (2'), target nuclei (3) and reaction products (4, 5, 6). The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction. FIG. 1(a) is a capture process and FIG. 1(b) is a reaction with two fragments.

higher with decreasing energy than the cross section of a usual, direct (first order) reaction since the huge exponential drop in the cross section (6) with decreasing energy disappears from the cross section of process (1).

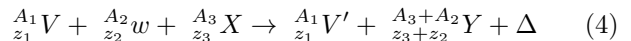
The disappearance of the exponential energy dependent factor in (6) means that due to impurity assisted reactions (1) the extra high temperature needed to ignite nuclear fusion in plasma may be appreciably reduced.

A less precise picture, still abiding the rules of second order time-independent perturbation calculation of quantum mechanics, can help to understand the effect with the aid of graphs (Fig. 1). The physics behind the calculation may be interpreted as follows. The Coulomb interaction between particles ${}_{z_1}^{A_1}V$ and protons mixes (an intermediate) state of the proton of large momentum to the initially slow protonic state with a small amplitude while particle ${}_{z_1}^{A_1}V$ is recoiled. Thus the Coulomb interaction pushes the protons (virtually) into an intermediate state. In this state protons have large enough (virtual) momentum to get over the Coulomb repulsion of nuclei ${}_{z_3}^{A_3}X$ and so they may be captured by the nuclei ${}_{z_3}^{A_3}X$ due to strong interaction to create a ${}_{z_3+1}^{A_3+1}Y$ nucleus. The particles (impurities) ${}_{z_1}^{A_1}V$ (initial) and ${}_{z_1}^{A_1}V'$ (final) assist the process only.

The virtual momentum of the intermediate state can be determined in the following way. Energy and momentum conservations determine the wave-vectors $\mathbf{k}_{1'}$ and \mathbf{k}_4 , of particles 1' and 4, respectively, as $\mathbf{k}_{1'} = -\mathbf{k}_4$ and $|\mathbf{k}_{1'}| = |\mathbf{k}_4| = k_0$ with $k_0 = \hbar^{-1}\sqrt{2m_0a_{14}\Delta}$. Here

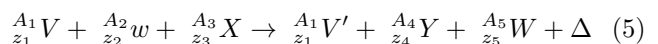
$m_0c^2 = 931.494 \text{ MeV}$ is the atomic energy unit and a_{14} is determined by (8). (The initial momenta and kinetic energies are assumed to be negligible.) Because of momentum conservation in Coulomb scattering of plane waves the wave vector $\mathbf{k}_{2'}$ of particle 2' is determined as $\mathbf{k}_{2'} = -\mathbf{k}_{1'}$, i.e. $|\mathbf{k}_{2'}| = k_0$ too. Consequently $|\mathbf{k}_{2'}|$ is large enough for particle 2' to effectively overcome the Coulomb repulsion. (For the details of the rigorous calculations see in Section III.)

A generalization of (1) can be the reaction



that will be briefly discussed to draw conclusions as to the possible modification of appropriate fuels of nuclear fusion reactors by impurity assisted reactions.

As a further generalization the reaction



with two final fragments is also considered. The impurity assisted $d(d, n){}_2^3\text{He}$, $d(d, p)t$, $d(t, n){}_2^4\text{He}$ and ${}_2^3\text{He}(d, p){}_2^4\text{He}$ reactions are numerically investigated and their rate and power densities are also determined.

In Section II. the essential role of the Coulomb factor is discussed. Section III. is devoted to the discussion of the model. The change of the wavefunction in the nuclear range due to the impurity is determined. The transition probability per unit time, cross section, rate and power densities of impurity assisted $p + d \rightarrow {}^3\text{He}$ reaction, which is the simplest impurity assisted proton capture reaction in an atomic-atom ionic gas mix, are given. The cross section of impurity assisted reactions with two final fragments are determined and the affect of plasma-wall interaction on the process is also considered. In Section IV. the rate and power densities in a $p - d - Xe$ atomic atom-ionic gas mix are calculated numerically. Section V. is a partial overview of some other impurity assisted nuclear reactions and gives account of the estimated power densities of the impurity assisted $d(d, n){}_2^3\text{He}$, $d(d, p)t$, $d(t, n){}_2^4\text{He}$, ${}_2^3\text{He}(d, p){}_2^4\text{He}$, ${}_3^6\text{Li}(p, \alpha){}_2^3\text{He}$, ${}_3^7\text{Li}(p, \alpha){}_2^4\text{He}$, ${}_4^9\text{Be}(p, \alpha){}_3^6\text{Li}$, ${}_4^9\text{Be}(p, d){}_4^8\text{Be}$, ${}_4^9\text{Be}(\alpha, n){}_6^{12}\text{C}$, ${}_5^{10}\text{B}(p, \alpha){}_4^7\text{Be}$ and ${}_5^{11}\text{B}(p, \alpha){}_4^8\text{Be}$ reactions. Section VI. is a Summary.

II. COULOMB FACTOR

The cross section (σ) of usual fusion reactions between particles 2 and 3 reads as [5]

$$\sigma(E) = S(E) \exp[-2\pi\eta_{23}(E_l)]/E, \quad (6)$$

where E is the energy taken in the center of mass (CM) coordinate system.

$$\eta_{23} = z_2 z_3 \alpha_f \frac{a_{23} m_0 c}{\hbar |\mathbf{k}|} = z_2 z_3 \alpha_f \sqrt{a_{23} \frac{m_0 c^2}{2E}} \quad (7)$$

is the Sommerfeld parameter. Here \mathbf{k} is the wave number vector of the particles 2 and 3 in their relative motion, \hbar is the reduced Planck-constant, c is the velocity of light in vacuum and

$$a_{jk} = \frac{A_j A_k}{A_j + A_k} \quad (8)$$

is the reduced mass number of particles j and k of mass numbers A_j and A_k and rest masses $m_j = A_j m_0$, $m_k = A_k m_0$. $m_0 c^2 = 931.494 \text{ MeV}$ is the atomic energy unit, α_f is the fine structure constant. The cross section (6) can be derived applying an approximate form

$$\varphi_{Cb,a}(\mathbf{r}) = f_{23}(k[E]) \exp(i\mathbf{k} \cdot \mathbf{r}) / \sqrt{V} \quad (9)$$

of the Coulomb solution $\varphi_{Cb}(\mathbf{r})$ [6] valid in the nuclear volume that produces the same $|\varphi(\mathbf{0})_{Cb,a}|^2 = f_{23}^2/V$ contact probability density as $\varphi_{Cb}(\mathbf{r})$. Here $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_3$ is the relative coordinate of particles 2 and 3 of coordinate \mathbf{r}_2 and \mathbf{r}_3 , and $f_{23}(k)$ is the Coulomb factor

$$f_{23} = \left| e^{-\pi\eta_{23}/2} \Gamma(1 + i\eta_{23}) \right| = \sqrt{\frac{2\pi\eta_{23}}{\exp(2\pi\eta_{23}) - 1}} \quad (10)$$

corresponding to particles 2 and 3.

The cross section is proportional to f_{23}^2 and one can show that the exponential factor in (6) comes from $f_{23}^2(E)$. Thus the smallness of rate at low energies is the consequence of $f_{23}^2(E)$ becoming very small at lower energies. So the magnitude of the Coulomb factor $f_{23}(E)$ is crucial from the point of view of magnitude of the cross section.

III. MODEL OF IMPURITY ASSISTED NUCLEAR REACTIONS IN AN ATOMIC-ATOM IONIC GAS MIX

A. Change of wavefunction in nuclear range

We focus on the modification of nuclear reactions in a plasma. At first capture processes are dealt with. It is supposed that all components are in atomic, atom-ionic or fully-ionized state while the necessary number of electrons required for electric neutrality are also present.

Let us take three screened charged particles of rest masses m_j ($j = 1, 2, 3$). Particles are heavy and have nuclear charges $z_j e$ with e the elementary charge and z_j the charge number. (The coupling strength $e^2 = \alpha_f \hbar c$.) The total Hamiltonian which describes this 3-body system is

$$H_{tot} = H_1 + H_{23} + V_{Cb}(1, 2) + V_{Cb}(1, 3), \quad (11)$$

where $H_1 = H_{kin,1}$ is the Hamiltonian of particle 1 which is considered to be free ($H_{kin,j}$ denotes the kinetic Hamiltonian of particle j),

$$H_{23} = H_{kin,2} + H_{kin,3} + V_{Cb}(1, 3) \quad (12)$$

is the Hamiltonian of particles 2 and 3. Their nuclear reaction will be discussed later. Here and in (11) $V_{Cb}(j, k)$ denotes screened Coulomb interaction between particles j and k with screening parameter $q_{sc,jk}$ and of form in the coordinate representation

$$V_{Cb}(j, k) = \frac{z_j z_k e^2}{2\pi^2} \int \frac{\exp(i\mathbf{q}\mathbf{r}_{jk})}{q^2 + q_{sc,jk}^2} d\mathbf{q}, \quad (13)$$

where $\mathbf{r}_{jk} = \mathbf{r}_j - \mathbf{r}_k$ is the relative coordinate between particles j and k of coordinate \mathbf{r}_j and \mathbf{r}_k . $H_{kin,2}$ and $H_{kin,3}$ are the kinetic Hamiltonians of particles 2 and 3.

It is supposed that stationary solutions $|1\rangle$ and $|2, 3\rangle_{sc}$ of energy eigenvalues E_1 and E_{23} of the stationary Schrödinger equations $H_1|1\rangle = E_1|1\rangle$ with E_1 the kinetic energy of particle 1 and $H_{23}|2, 3\rangle_{sc} = E_{23}|2, 3\rangle_{sc}$ with $E_{23} = E_{CM} + E_{rel}$ are known. Here E_{CM} and E_{rel} are the energies attached to the center of mass (CM) and relative motions of particles 2 and 3. Thus H_{tot} can be written as $H_{tot} = H_0 + H_{Int}$ with $H_0 = H_1 + H_{23}$ as the unperturbed Hamiltonian and

$$H_{Int} = V_{Cb}(1, 2) + V_{Cb}(1, 3) \quad (14)$$

as the interaction Hamiltonian (perturbation). The stationary solution $|1, 2, 3\rangle_{sc}$ of $H_0|1, 2, 3\rangle_{sc} = E_0|1, 2, 3\rangle_{sc}$ with $E_0 = E_1 + E_{23}$ can be written as $|1, 2, 3\rangle_{sc} = |1\rangle|2, 3\rangle_{sc}$ which is the direct product of states $|1\rangle$ and $|2, 3\rangle_{sc}$. The states $|1, 2, 3\rangle_{sc}$ form an orthonormal complete system.

The approximate solution of $H_{tot}|1, 2, 3\rangle_{sc} = E_0|1, 2, 3\rangle_{sc}$ in the screened case is obtained with the aid of standard time independent perturbation calculation [4] and the first order approximation is expanded in terms of the complete system. The terms which differ from the initial state and which are considered to be intermediate from the point of view of the next step of perturbation calculation taking into account strong interaction will be called intermediate states.

The solution of $H_{23}|2, 3\rangle = E_{23}|2, 3\rangle$ in the unscreened case is known and the coordinate representation $\langle \mathbf{R}, \mathbf{r} | 2, 3 \rangle = \varphi_{23}(\mathbf{R}, \mathbf{r})$ of $|2, 3\rangle$ has the form

$$\varphi_{23}(\mathbf{R}, \mathbf{r}) = \frac{e^{i\mathbf{K}\mathbf{R}}}{\sqrt{V}} \varphi_{Cb}(\mathbf{r}), \quad (15)$$

where \mathbf{K} , $\mathbf{R} = (m_2\mathbf{r}_2 + m_3\mathbf{r}_3) / (m_2 + m_3)$ and $\mathbf{r} = \mathbf{r}_{23}$ are wave vector of the CM motion, CM and relative coordinate of particles 2 and 3, respectively, V denotes the volume of normalization and

$$\varphi_{Cb}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} f(\mathbf{k}, \mathbf{r}) / \sqrt{V} \quad (16)$$

is the unscreened Coulomb solution [6]. Here \mathbf{k} is the wave number vector of the particles 2 and 3 in their relative motion and $f(\mathbf{k}, \mathbf{r}) = e^{-\pi\eta_{23}/2} \times \Gamma(1 + i\eta_{23}) \times {}_1F_1(-i\eta_{23}, 1; i[kr - \mathbf{k}\cdot\mathbf{r}])$, where ${}_1F_1$ is the confluent hypergeometric function, Γ is the Gamma function and η_{23} is the Sommerfeld parameter, furthermore

$$E_{rel} = \frac{\hbar^2 \mathbf{k}^2}{2m_0 a_{23}} \quad (17)$$

and

$$E_{CM} = \frac{\hbar^2 \mathbf{K}^2}{2m_0(A_2 + A_3)}. \quad (18)$$

The two important limits of the eigenstate $|2, 3\rangle_{sc}$ in the case of screened Coulomb potential are the solution in the nuclear volume and the solution in the screened regime. (In the screened case the coordinate representation $\langle \mathbf{R}, \mathbf{r} | 2, 3\rangle_{sc}$ is denoted by $\varphi_{23}(\mathbf{R}, \mathbf{r})_{sc}$.)

In the screened case and in the nuclear volume the approximate form (9) ($\varphi_{Cb,a}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} f_{23}(k)/\sqrt{V}$) of the (unscreened) Coulomb solution (16) may be used. Here $f_{23}(k)$ is the appropriate Coulomb factor corresponding to particles 2 and 3. Thus

$$\varphi_{23}(\mathbf{R}, \mathbf{r}, nucl)_{sc} = \frac{e^{i\mathbf{K}\mathbf{R}}}{\sqrt{V}} \varphi_{Cb,a}(\mathbf{k}, \mathbf{r}) \quad (19)$$

is used in the range of the nucleus and in the calculation of the nuclear matrix-element.

The other important limit of $|2, 3\rangle_{sc}$ is the screened (outer) range where $\varphi_{23}(\mathbf{R}, \mathbf{r})_{sc}$ becomes

$$\varphi_{23}(\mathbf{R}, \mathbf{r}, out)_{sc} = \frac{e^{i\mathbf{K}\mathbf{R}} e^{i\mathbf{k}\mathbf{r}}}{V} \quad (20)$$

of energy eigenvalue also $E_{23} = E_{CM} + E_{rel}$. It is used in the calculation of the Coulomb matrix element.

From the point of view of the processes investigated the initial state of negligible wave number and energy ($E_0 = E_i = 0$) can be written as $\varphi_i = V^{-3/2}$ for particles 1, 2 and 3 that are somewhere in the normalization volume. The intermediate states of particles 2 and 3 are determined by the wave number vectors \mathbf{K} and \mathbf{k} . In the case of the assisting particle 1 the intermediate state is a plane wave of wave number vector \mathbf{k}_1 .

The matrix elements $V_{Cb,\nu i}$ of the screened Coulomb potential between the initial and intermediate states are

$$V_{Cb(1,s)\nu i} = \frac{z_1 z_s e^2 (2\pi)^9}{2\pi^2 V^3} \delta(\mathbf{k}_1 + \mathbf{K}) \times \quad (21)$$

$$\times \frac{\delta(\mathbf{k} + a(s)\mathbf{k}_1)}{\mathbf{k}_1^2 + q_{sc,1s}^2}$$

where

$$a(s) = \frac{-A_3 \delta_{s,2} + A_2 \delta_{s,3}}{A_2 + A_3} \text{ and } s = 2, 3.$$

which according to standard time independent perturbation theory of quantum mechanics [4] determine the first order change of the wavefunction in the range $r \lesssim R_0$ (R_0 is the nuclear radius of particle 3) due to Coulomb perturbation as

$$\delta\varphi(\mathbf{r}) = \sum_{s=2,3} \delta\varphi(s, \mathbf{r}) \quad (22)$$

$$\delta\varphi(s, \mathbf{r}) = \int \int \frac{V_{Cb(1,s)\nu i}}{E_\nu - E_i} \frac{V}{(2\pi)^6} \times \quad (23)$$

$$\times e^{i(\mathbf{K}\mathbf{R} + \mathbf{k}_1\mathbf{r}_1)} \varphi_{Cb,a}(\mathbf{k}, \mathbf{r}) d\mathbf{K} d\mathbf{k},$$

where E_i and E_ν are the kinetic energies in the initial and intermediate states, respectively. The initial momenta and kinetic energies of particles 1, 2 and 3 are neglected ($E_i = 0$).

$$E_\nu(\mathbf{K}, \mathbf{k}) = \frac{\hbar^2 \mathbf{K}^2}{2m_0(A_2 + A_3)} + \frac{\hbar^2 \mathbf{k}^2}{2m_0 a_{23}} + \frac{\hbar^2 \mathbf{k}_1^2}{2m_0 A_1}, \quad (24)$$

Thus

$$\delta\varphi(s, \mathbf{r}) = z_1 z_s \alpha_f \frac{4\pi \hbar c}{V^{5/2}} \frac{e^{i(\mathbf{k}_1\mathbf{r}_1 - \mathbf{k}_1\mathbf{R})}}{\mathbf{k}_1^2 + q_{sc,1s}^2} \times \quad (25)$$

$$\times \frac{2m_0 a_{1s}}{\hbar^2 \mathbf{k}_1^2} [f_{23}(k) e^{i\mathbf{k}\mathbf{r}}]_{\mathbf{k}=a(s)\mathbf{k}_1}.$$

It can be seen that the arguments of $f_{23}(k)$ are $\frac{A_3}{A_2 + A_3} k_1$ and $\frac{A_2}{A_2 + A_3} k_1$, here $k_1 = |\mathbf{k}_1|$. Consequently, if particle 1 obtains large kinetic energy, as is the case in the nuclear reaction, then the Coulomb factors $f_{23}(k)$ and the rate of the process too will considerably increase. Furthermore, $\delta\varphi(\mathbf{r})$, which causes the effect, is temperature independent. Up to this point the calculation and the results are nuclear reaction and nuclear model independent.

B. Transition probability per unit time and cross section of p-capture

Now we can calculate the rate of nuclear reaction due to the modification caused by particle 1. (The intermediate and final states of particle 1 are identical.)

The Hamiltonian $V_{st}(2, 3)$ of strong interaction which is responsible for nuclear reactions between particles 2 and 3 is

$$V_{st}(2, 3) = -V_0 \text{ if } |\mathbf{r}_{23}| = |\mathbf{r}| \leq b \text{ and} \quad (26)$$

$$V_{st}(2, 3) = 0 \text{ if } |\mathbf{r}_{23}| = |\mathbf{r}| > b.$$

We take $V_0 = 25 \text{ MeV}$ and $b = 2 \times 10^{-13} \text{ cm}$ [7] in the case of pd reaction.

The final state of particle 1 is a plane wave of wave number \mathbf{k}_1 and of kinetic energy $E_{1f} = \hbar^2 \mathbf{k}_1^2 / (2m_1)$. The final state of the captured proton has the form

$$\varphi_4(\mathbf{R}, \mathbf{r}) = e^{i\mathbf{k}_4 \cdot \mathbf{R}} \Phi_f(\mathbf{r}) / \sqrt{V}, \quad (27)$$

where $\Phi_f(\mathbf{r})$ is the final nuclear state of the proton in particle 4 and \mathbf{k}_4 is the wave vector of particle 4. It has kinetic energy $E_{4f} = \hbar^2 \mathbf{k}_4^2 / (2m_4)$. The Weisskopf approximation is used, i.e. $\Phi_f(\mathbf{r}) = \Phi_{fW}(\mathbf{r})$ with

$$\Phi_{fW}(\mathbf{r}) = \sqrt{\frac{3}{4\pi R_0^3}} \quad (28)$$

if $r \leq R_0$, where R_0 is the nuclear radius, and $\Phi_{fW}(\mathbf{r}) = 0$ for $r > R_0$.

$V_{st,f\nu}$ is the matrix element of the potential of the strong interaction between intermediate $\left(e^{i\mathbf{K}\mathbf{R}}\varphi_{Cb,a}(\mathbf{k},\mathbf{r})/\sqrt{V}\right)$ and final $\left(e^{i\mathbf{k}_4\cdot\mathbf{R}}\Phi_f(\mathbf{r})/\sqrt{V}\right)$ states.

Since $\Phi_{fW}(\mathbf{r})$ and $V_{st}(\mathbf{r})$ both have spherical symmetry the spherical term $\sin(kr)/(kr)$ remains from $e^{i\mathbf{k}\cdot\mathbf{r}}$, which is present in $\varphi_{Cb,a}(\mathbf{k},\mathbf{r})$, in the nuclear matrix-element. With the aid of the above wave function and the $b = R_0$ assumption

$$V_{st,f\nu}^W = -V_0 \frac{\sqrt{12\pi R_0}}{k} f_{23}(k) H(k) \frac{(2\pi)^3}{V^{3/2}} \delta(\mathbf{K} - \mathbf{k}_4) \quad (29)$$

where

$$H(k) = \int_0^1 \sin(kR_0x)xdx. \quad (30)$$

According to standard time independent perturbation theory of quantum mechanics the transition probability per unit time $\left(W_{fi}^{(2)}\right)$ of the process can be written as

$$W_{fi}^{(2)} = \frac{2\pi}{\hbar} \int \int \left|T_{fi}^{(2)}\right|^2 \delta(E_f - \Delta) \frac{V^2}{(2\pi)^6} d\mathbf{k}_1 d\mathbf{k}_4 \quad (31)$$

with

$$T_{fi}^{(2)} = \int \int \sum_{s=2,3} \frac{V_{st,f\nu} V_{Cb}(1,s)\nu_i}{E_\nu - E_i} \frac{V^2}{(2\pi)^6} d\mathbf{K} d\mathbf{k}, \quad (32)$$

Collecting everything obtained above, substituting (32) into (31), neglecting $q_{sc,jk}^2$ since $q_{sc,jk}^2 \ll \mathbf{k}_1^2 = \mathbf{k}_0^2$ with

$$k_0 = \hbar^{-1} \sqrt{2m_0 a_{14} \Delta} \quad (33)$$

determined by energy conservation one can calculate $W_{fi}^{(2)}$. The cross section $\sigma_{23}^{(2)}$ of the process is defined as

$$\sigma_{23}^{(2)} = \frac{N_1 W_{fi}^{(2)}}{\frac{v_{23}}{V}} \quad (34)$$

where N_1 is the number of particles 1 in the normalization volume V and v_{23}/V is the flux of particle 2 of relative velocity v_{23} .

$$v_{23} \sigma_{23}^{(2)} = n_1 S_{pd} \quad (35)$$

where $n_1 = N_1/V$ is the number density of particles 1 and

$$S_{pd} = 24\pi^2 \sqrt{2} c R_0 \frac{z_1^2 \alpha_f^2 V_0^2 (\hbar c)^4}{\Delta^{9/2} (m_0 c^2)^{3/2}} \times \quad (36)$$

$$\times \frac{(A_2 + A_3)^2}{a_{14}^{7/2}} [F(2) + F(3)]^2.$$

with

$$F(s) = \frac{z_s a_{1s}}{A_3 \delta_{s,2} + A_2 \delta_{s,3}} f_{23}[a(s)k_0] H[a(s)k_0]. \quad (37)$$

The magnitude of quantities $f_{23}[a(s)k_0]$, $s = 2, 3$ mainly determines the magnitude of the rate and power densities.

C. Rate and power densities

The rate dN_{pd}/dt in the whole volume V can be written as

$$\frac{dN_{pd}}{dt} = N_3 \Phi_{23} \sigma_{23}^{(2)}, \quad (38)$$

where $\Phi_{23} = n_2 v_{23}$ is the flux of particles 2 with n_2 their number density ($n_2 = N_2/V$) and N_3 is the number of particles 3 in the normalization volume. The rate density $r_{pd} = dn_{pd}/dt = V^{-1} dN_{pd}/dt$ of the process can be written as

$$r_{pd} = \frac{dn_{pd}}{dt} = n_3 n_2 n_1 S_{pd}, \quad (39)$$

where n_3 is the number density of particles 3 ($n_3 = N_3/V$). The power density is defined as

$$p_{pd} = \Delta \frac{dn_{pd}}{dt} = n_1 n_2 n_3 P_{pd} \quad (40)$$

with $P_{pd} = S_{pd} \Delta$. The rate and power densities (dn_{pd}/dt and p_{pd}) are temperature independent.

D. Cross section of reactions with two final fragments in long wavelength approximation

In the case of reactions with two final fragments (see Fig.1b and (5)) the nuclear matrix element can be derived from the $S(E)$ quantity of (6), i.e. in long wavelength approximation from $S(0)$ which is the astrophysical factor at $E = 0$. It can be done in the following manner.

Calculating the transition probability per unit time $W_{fi}^{(1)}$ of the usual (first order) process in standard manner

$$W_{fi}^{(1)} = \int \frac{2\pi}{\hbar} |V_{st,fi}|^2 \delta(E_f - \Delta) \frac{V}{(2\pi)^3} d\mathbf{k}_f$$

where \mathbf{k}_f is the relative wave number of the two fragments of rest masses $m_4 = m_0 A_4$, $m_5 = m_0 A_5$ and atomic numbers A_4, A_5 , $E_f = \hbar^2 \mathbf{k}_f^2 / (2m_0 a_{45})$ is the sum of their kinetic energy and the nuclear matrix element is $V_{st,fi}$ having the form $|V_{st,fi}| = f_{23}(k_i) h_{fi}/V$. Here $f_{23}(k_i)$ is the Coulomb factor of the initial particles 2 and 3 with k_i the magnitude of their relative wave number vector \mathbf{k}_i . (The Coulomb factor $f_{45}(k_f) \approx 1$ of the final particles 4 and 5 with k_f the magnitude of their

relative wave number vector \mathbf{k}_f .) It is supposed that h_{fi} does not depend on \mathbf{k}_i and \mathbf{k}_f namely the long wavelength approximation is used. In this case the product of the relative velocity v_{23} of the initial particles 2, 3 and the cross section σ_{23} is

$$v_{23}\sigma_{23}^{(1)} = \frac{|h_{fi}|^2}{\pi\hbar} f_{23}^2(k_i) \frac{(m_0 a_{45})^{3/2}}{\hbar^3} \sqrt{2\Delta}. \quad (41)$$

On the other hand, $v_{23}\sigma_{23}^{(1)}$ is expressed with the aid of (6) and $v_{23} = \sqrt{2E/(m_0 a_{23})}$. From the equality of the two kinds of $v_{23}\sigma_{23}^{(1)}$ one gets

$$|h_{fi}|^2 = \frac{(\hbar c)^4 S(0)}{z_2 z_3 \alpha_f (m_0 c^2)^{5/2} \sqrt{2\Delta} a_{45}^{3/2} a_{23}}. \quad (42)$$

In the case of the impurity assisted, second order process $|V_{st,f\nu}| = f_{23}(k) |h_{fi}| (2\pi)^3 \delta(\mathbf{K} - \mathbf{K}_f) / V^2$ where \mathbf{K}_f and \mathbf{k}_f are the final wave number vectors attached to *CM* and relative motions of the two final fragments, particles 4 and 5. \mathbf{k}_f appears in E_f in the energy Dirac-delta. Repeating the calculation of the transition probability per unit time of the impurity assisted, second order process applying the above expression of $|V_{st,f\nu}|$ one gets

$$v_{23}\sigma_{23}^{(2)} = n_1 S'_{reaction'}, \quad (43)$$

where

$$S'_{reaction'} = \frac{8\alpha_f^2 z_1^2 S(0)c}{a_{23} a_{123}^3 m_0 c^2} \left(\frac{\hbar c}{\Delta}\right)^3 I \quad (44)$$

with

$$I = \int_0^1 \left(\sum_{s=2,3} \frac{z_s a_{1s} \sqrt{A_s}}{\sqrt{e^{b_{23} A_s \frac{1}{x}} - 1}} \right)^2 \frac{\sqrt{1-x^2}}{x^7} dx. \quad (45)$$

Here $b_{23} = 2\pi z_2 z_3 \alpha_f \sqrt{m_0 c^2 / (2a_{123} \Delta)}$ with $a_{123} = A_1(A_2 + A_3) / (A_1 + A_2 + A_3)$. In the index '*reaction*' the reaction resulting the two fragments will be marked.

E. Cross section of reactions with two final fragments beyond long wavelength approximation

If $|h_{fi}|$ has \mathbf{k}_1 dependence then it is manifested through the relative energy (E) dependence of the astrophysical factor $[S(E)]$ and it can be expressed as

$$|h_{fi}(k_1)| = \sqrt{\frac{(\hbar c)^4 S[E(k_1)]}{z_2 z_3 \alpha_f (m_0 c^2)^{5/2} \sqrt{2\Delta} a_{45}^{3/2} a_{23}}} \quad (46)$$

where

$$E(k_1) = \left[\frac{\hbar^2 \mathbf{k}^2}{2m_0 a_{23}} \right]_{\mathbf{k}=\mathbf{a}(s)\mathbf{k}_1} = \frac{\hbar^2 a^2(s) k_1^2}{2m_0 a_{23}}. \quad (47)$$

Consequently

$$S'_{reaction'} = \frac{8\alpha_f^2 z_1^2 S(0)c}{a_{23} a_{123}^3 m_0 c^2} \left(\frac{\hbar c}{\Delta}\right)^3 J \quad (48)$$

with

$$J = \int_0^1 \left(\sum_{s=2,3} \frac{z_s a_{1s} \sqrt{A_s} \sqrt{\frac{S[E(s,x)]}{S(0)}}}{\sqrt{e^{b_{23} A_s \frac{1}{x}} - 1}} \right)^2 \times \frac{\sqrt{1-x^2}}{x^7} dx. \quad (49)$$

The argument of $S(E)$ in the integrand is

$$E(s, x) = \Delta \frac{a_{123}}{a_{23}} a^2(s) x^2. \quad (50)$$

F. Atomic atom-ionic gas mix and wall interaction

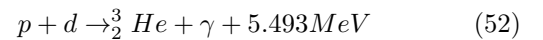
It is plausible to extend the investigation to the possible consequence of plasma-wall interaction. The role of particle 1 is played by the wall which is supposed to be a solid (metal) from atoms with nuclei of charge and mass numbers z_1 and A_1 . For initial state a Bloch-function of the form

$$\varphi_{\mathbf{k}_{1,i}}(\mathbf{r}_1) = \frac{1}{\sqrt{N_1}} \sum_{\mathbf{L}} e^{i\mathbf{k}_{1,i} \cdot \mathbf{L}} a(\mathbf{r}_1 - \mathbf{L}), \quad (51)$$

is taken, that is localized around all of the lattice points [8]. Here \mathbf{r}_1 is the coordinate, $\mathbf{k}_{1,i}$ is wave number vector of the first Brillouin zone (*BZ*) of the reciprocal lattice, $a(\mathbf{r}_1 - \mathbf{L})$ is the Wannier-function, which is independent of $\mathbf{k}_{1,i}$ within the *BZ* and is well localized around lattice site \mathbf{L} . N_1 is the number of lattice points of the lattice of particles 1. Repeating the transition probability per unit time and cross section calculation applying (51) (after a lengthy calculation which is omitted here) it is obtained that cross section results (formulae (35), (36) and (37) in case of proton capture, (44), (45) and (48), (49) in case of reactions with two final fragments) remain unchanged and $n_1 = N_{1c}/v_c$, where v_c is the volume of elementary cell of the solid and N_{1c} is the number of particles 1 in the elementary cell.

IV. RATE AND POWER DENSITIES IN A $p-d-Xe$ ATOMIC ATOM-IONIC GAS MIX

Reaction



is not suitable for energy production since its cross section (the $S(0)$ value, see [5]) is rather small compared with other candidate reactions and only a minor part of the reaction energy $\Delta = 5.493\text{ MeV} = 8.800 \times 10^{-13}\text{ J}$

is taken away by ${}^3\text{He}$ ($E_{3\text{He}} = \Delta^2 / (6m_0c^2) = 5.4 \text{ keV}$) and the main part $E_\gamma = 5.488 \text{ MeV}$ is taken away by γ radiation which is difficult to convert to heat. However, in reaction (3) the reaction energy is taken away by particles ${}^3_2\text{He}$ and ${}^{A_1}_{z_1}\text{V}'$ as their kinetic energy that they can lose in a very short range to their environment converting the reaction energy efficiently into heat. Therefore the direct observation of ${}^3_2\text{He}$ and ${}^{A_1}_{z_1}\text{V}'$ is hard.

The rate (dn_{pd}/dt) and power ($\Delta dn_{pd}/dt$) densities of impurity assisted $p + d \rightarrow {}^3_2\text{He}$ reaction are determined by (39) and (40) with

$$S_{pd} = 1.89 \times 10^{-53} z_1^2 \text{ cm}^6 \text{ s}^{-1}, \quad (53)$$

where z_1 is the charge number of the assisting nucleus and with

$$P_{pd} = 1.66 \times 10^{-65} z_1^2 \text{ cm}^6 \text{ W}, \quad (54)$$

respectively. Taking $z_1 = 54$ (Xe) and $n_1 = n_2 = n_3 = 2.65 \times 10^{20} \text{ cm}^{-3}$ (n_1, n_2 and n_3 are the number densities of Xe, p and d , i.e. particles 1, 2 and 3) one gets for rate and power densities considerable values:

$$r_{pd} = dn_{pd}/dt = 1.02 \times 10^{12} \text{ cm}^{-3} \text{ s}^{-1} \quad (55)$$

and

$$p_{pd} = \Delta dn_{pd}/dt = 0.901 \text{ W cm}^{-3} \quad (56)$$

. If the impurity is Hg or U then the above numbers must be multiplied by 2.2 or 2.9, respectively.

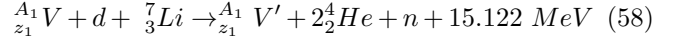
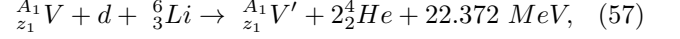
One must emphasize that both rate and power densities (dn_{pd}/dt and p_{pd}) are temperature independent. It must be mentioned too that the effect is not affected by Coulomb screening and the only condition is that the participants must be in atomic or in atom-ionic state. This requirement and the temperature independence of dn_{pd}/dt and p_{pd} greatly weaken the necessary conditions.

V. OTHER IMPURITY ASSISTED NUCLEAR REACTIONS

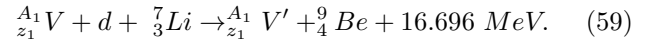
Now let us consider the impurity assisted proton captures (1) in general. The reaction energy Δ is the difference between the sum of the initial and final mass excesses, i.e. $\Delta = \Delta_p + \Delta_{A_3, z_3} - \Delta_{A_3+1, z_3+1}$. Since particle 1 assists the nuclear reaction its rest mass does not change. $\Delta_p, \Delta_{A_3, z_3}$ and Δ_{A_3+1, z_3+1} are mass excesses of proton, ${}^{A_3}_{z_3}\text{X}$ and ${}^{A_3+1}_{z_3+1}\text{Y}$ nuclei, respectively [9]. Moreover, the capture reaction may be extended to the impurity assisted capture of particles ${}^{A_2}_{z_2}w$ (see reaction (4)), e.g. the capture of deuteron (d), triton (t), ${}^3\text{He}$, ${}^4\text{He}$, etc.. In this case $\Delta = \Delta_{A_2, z_2} + \Delta_{A_3, z_3} - \Delta_{A_3+A_2, z_3+z_2}$. $\Delta_{A_2, z_2}, \Delta_{A_3, z_3}$ and $\Delta_{A_3+A_2, z_3+z_2}$ are the corresponding mass excesses. The mechanism discovered makes also possible reaction (5) with conditions $A_2 + A_3 = A_4 + A_5$ and $z_2 + z_3 = z_4 + z_5$. In this case $\Delta = \Delta_{A_2, z_2} + \Delta_{A_3, z_3} -$

$\Delta_{A_4, z_4} - \Delta_{A_5, z_5}$ where Δ_{A_j, z_j} are the corresponding mass excesses. Investigating the mass excess data [9] one can recognize that in the case of processes (1), (4) and (5) the number of energetically allowed reactions is large, their usefulness from the point of view of energy production is mainly determined by the magnitude of the numerical values of the quantities f_{23} belonging to the particular reaction.

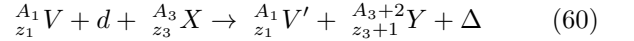
Impurity (${}^{A_1}_{z_1}\text{V}$) assisted $d - \text{Li}$ reactions may take place with ${}^6_3\text{Li}$ and ${}^7_3\text{Li}$ isotopes:



and



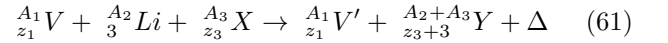
If there are deuterons present then the



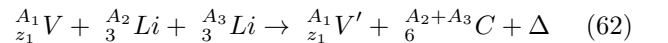
impurity assisted d capture processes (see e.g. (59) and the ${}^{A_1}_{z_1}\text{V} + d + d \rightarrow {}^{A_1}_{z_1}\text{V}' + {}^4_2\text{He} + 23.847 \text{ MeV}$ reaction), furthermore the ${}^{A_1}_{z_1}\text{V} + d + d \rightarrow {}^{A_1}_{z_1}\text{V}' + n + {}^3_2\text{He} + 3.269 \text{ MeV}$, ${}^{A_1}_{z_1}\text{V} + d + d \rightarrow {}^{A_1}_{z_1}\text{V}' + p + t + 4.033 \text{ MeV}$ impurity assisted dd reactions may also take place where the energy of the reaction is carried by particles ${}^{A_1}_{z_1}\text{V}'$, ${}^{A_3+2}_{z_3+1}\text{Y}$ and ${}^{A_1}_{z_1}\text{V}'$, ${}^4_2\text{He}$, which have momentum of equal magnitude but opposite direction, by particles ${}^{A_1}_{z_1}\text{V}'$, n and ${}^3_2\text{He}$ and by particles ${}^{A_1}_{z_1}\text{V}'$, p and t , respectively.

The results of $S_{\text{reaction}'}$ and power density calculations of some Xe assisted reactions in long wavelength approximation and with $n_1 = n_2 = n_3 = 2.65 \times 10^{20} \text{ cm}^{-3}$ can be found in Table I. From the point of view of rate and power densities the screening of the Coulomb potential is not essential ($k_0 \gg q_{sc}$) consequently the above reactions bring up the possibility of a quite new type of apparatus since the processes need atomic state of participant materials only, i.e. need much lower temperature compared to the working temperature of fusion power stations planned to date.

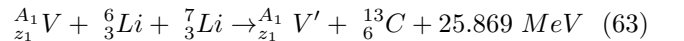
If there is Li present then



impurity assisted Li capture reactions may happen too. Let us examine the impurity assisted



Li capture reactions and as an example let us take $z_2 = z_3 = 3$, $A_2 = 6$, $A_3 = 7$, $A_2 + A_3 = A_4 = 13$, that corresponds to the



reaction. Taking $A_1 = 130$, $\eta_{23}(\frac{7}{13}k_0) = 0.487$, $\eta_{23}(\frac{6}{13}k_0) = 0.568$ and $f_{23}(\frac{7}{13}k_0) = 0.388$ and

Reaction	$S(0)$	$S'_{Reaction'}$	Δ	$p'_{Reaction'}$
$d(d, n) {}_2^3\text{He}$	0.055	1.01×10^{-48}	3.269	9.82
$d(d, p)t$	0.0571	1.10×10^{-48}	4.033	13.2
$d(t, n) {}_2^4\text{He}$	11.7	1.06×10^{-46}	17.59	5.57×10^3
${}^3_2\text{He}(d, p) {}^4_2\text{He}$	5.9	1.51×10^{-48}	18.25	82.6
${}^6_3\text{Li}(p, \alpha) {}^3_2\text{He}$	2.97	1.99×10^{-49}	4.019	2.38
${}^7_3\text{Li}(p, \alpha) {}^4_2\text{He}$	0.0594	3.85×10^{-51}	17.347	0.199
${}^9_4\text{Be}(p, \alpha) {}^6_3\text{Li}$	17.	1.79×10^{-49}	2.126	1.13
${}^9_4\text{Be}(p, d) {}^8_4\text{Be}$	17.	1.66×10^{-49}	0.56	0.277
${}^9_4\text{Be}(\alpha, n) {}^{12}_6\text{C}$	2.5×10^3	6.22×10^{-51}	5.701	0.106
	$6. \times 10^5$	1.49×10^{-48}	5.701	25.4
${}^{10}_5\text{B}(p, \alpha) {}^7_4\text{Be}$	4	1.04×10^{-50}	1.145	0.0356
	$2. \times 10^3$	5.21×10^{-48}	1.145	17.8
${}^{11}_5\text{B}(p, \alpha) {}^8_4\text{Be}$	187	5.16×10^{-49}	8.59	13.2

TABLE I: $S(0)$ is the astrophysical factor at $E = 0$ in $MeVb$ [5], [10]. $S'_{Reaction'}$ (in cm^6s^{-1}) is calculated using (44) with (45) taking $z_1 = 54$ (Xe), Δ is the energy of the reaction in MeV and $p'_{Reaction'} = \Delta n_1 n_2 n_3 S'_{Reaction'}$ is the power density in Wcm^{-3} that is calculated with $n_1 = n_2 = n_3 = 2.65 \times 10^{20} cm^{-3}$. In the case of ${}^9_4\text{Be}(\alpha, n) {}^{12}_6\text{C}$ and ${}^{10}_5\text{B}(p, \alpha) {}^7_4\text{Be}$ reactions the astrophysical factor $[S(E)]$ has strong energy dependence therefore the calculation was carried out with two characteristic values of $S(E)$.

$f_{23}(\frac{6}{13}k_0) = 0.322$ (see (7), (10), (33) and (36)). These numbers are very promising. The reactions ${}^{A_1}_{z_1}V + {}^6_3Li + {}^6_3Li \rightarrow {}^{A_1}_{z_1}V' + {}^{12}_6C + 28.174 MeV$, ${}^{A_1}_{z_1}V + {}^6_3Li + {}^6_3Li \rightarrow$

${}^{A_1}_{z_1}V' + {}^3_2He + 20.898 MeV$ and ${}^{A_1}_{z_1}V + {}^7_3Li + {}^7_3Li \rightarrow {}^{A_1}_{z_1}V' + {}^{14}_6C + 26.795 MeV$ may have importance too. (The list is incomplete.)

VI. SUMMARY

The consequences of impurities in nuclear fusion fuels of plasma state are discussed. According to calculations in certain cases second order processes may produce greatly higher fusion rate than the rate due to direct (first order) processes. In the examined problem it is found that Coulomb scattering of the fusible nuclei on the screened Coulomb potential of the impurity can diminish the hindering Coulomb factor between them. Since the second order process does not demand the matter to be in ionized state the assistance of impurities can allow to decrease significantly the plasma temperature which is determined only by the requirement that all components must be in atomic or atom-ionic state. The results suggest that, on the other hand, the density of the components has to be considerably increased. The effective influence of wall-gas mix interaction brings up the possible importance of gas mix-metal surface processes too. Promising new fuel mixes are also put forward. Based on these results it may be expected that search for new approach to energy production by nuclear fusion may be started.

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