Forbidden Nuclear Reactions

Péter Kálmán and Tamás Keszthelyi
Budapest University of Technology and Economics,
Institute of Physics, Budafoki út 8. F., H-1521 Budapest, Hungary
(Date textdate; Received textdate; Revised textdate; Accepted textdate; Published textdate)

Exothermal nuclear reactions which become forbidden due to Coulomb repulsion in the $E \rightarrow 0$ limit ($\lim_{E\rightarrow 0} \sigma(E) = 0$) are investigated. ($\sigma(E)$ is the cross section and E is the center of mass energy.) It is found that *any perturbation* may mix states with small but finite amplitude to the initial state resulting finite cross section (and rate) of the originally forbidden nuclear reaction in the $E \rightarrow 0$ limit. The statement is illustrated by modification of nuclear reactions due to impurities in plasma. The change of the wavefunction of reacting particles in nuclear range due to their Coulomb interaction with impurity is determined using standard time independent perturbation calculation of quantum mechanics. As an example, cross section, rate and power densities of impurity assisted nuclear *pd* reaction are numerically calculated. With the aid of astrophysical factors cross section and power densities of the impurity assisted $d(d,n)_2^3He$, $d(d,p)t$, $d(t,n)_2^4He$, $_2^3He(d,p)_2^4He$, $_3^6Li(p,\alpha)_2^3He$, $_3^6Li(d,\alpha)_2^4He$, $_3^7Li(p,\alpha)_2^4He$, $_4^9Be(p,\alpha)_3^6Li$, $_4^9Be(p,d)_4^8Be$, $_4^9Be(\alpha,n)_6^{12}C$, $_5^{10}B(p,\alpha)_4^7Be$ and $_5^{11}B(p,\alpha)_4^8Be$ reactions are also given. The affect of plasma-wall interaction on the process is considered too.

PACS numbers: 24.90.+d, 03.65.-w, 25.60.Pj
Keywords: nuclear reactions: specific reactions: general, quantum mechanics, fusion reactions

I. INTRODUCTION

The cross section (σ) of nuclear reactions between charged particles j and k of charge numbers z_j and z_k reads as [1]

$$\sigma(E) = S(E) \exp[-2\pi\eta_{jk}(E)]/E, \tag{1}$$

where $S(E)$ is the astrophysical S-factor and E is the kinetic energy taken in the center of mass (CM) coordinate system.

$$\eta_{jk} = z_j z_k \alpha_f \frac{a_{jk} m_0 c}{\hbar |\mathbf{k}|} = z_j z_k \alpha_f \sqrt{a_{jk} \frac{m_0 c^2}{2E}} \tag{2}$$

is the Sommerfeld parameter, where $\mathbf{k}$ is the wave number vector of particles j and k in their relative motion, $\hbar$ is the reduced Planck-constant, c is the velocity of light in vacuum and

$$a_{jk} = \frac{A_j A_k}{A_j + A_k} \tag{3}$$

is the reduced mass number of particles j and k of mass numbers A_j and A_k and rest masses $m_j = A_j m_0$, $m_k = A_k m_0$. $m_0 c^2 = 931.494\ MeV$ is the atomic energy unit, α_f is the fine structure constant.

The solution $\varphi_{jk}(\mathbf{R}, \mathbf{r})$ of the sationary Schrödinger equation

$$H_{jk}\varphi_{jk}(\mathbf{R}, \mathbf{r}) = E_{jk}\varphi_{jk}(\mathbf{R}, \mathbf{r}) \tag{4}$$

with

$$H_{jk} = -\frac{\hbar^2}{2m_0(A_j + A_k)}\nabla_{\mathbf{R}}^2 - \frac{\hbar^2}{2m_0 a_{jk}}\nabla_{\mathbf{r}}^2 + \frac{z_j z_k e^2}{|\mathbf{r}|} \tag{5}$$

is

$$\varphi_{jk}(\mathbf{R}, \mathbf{r}) = V^{-1/2} e^{i\mathbf{K}\mathbf{R}} \varphi_{Cb}(\mathbf{r}), \tag{6}$$

where $\mathbf{R} = (m_j \mathbf{r}_j + m_k \mathbf{r}_k)/(m_j + m_k)$ and $\mathbf{r} = \mathbf{r}_{jk} = \mathbf{r}_j - \mathbf{r}_k$ are CM and relative coordinate of particles j and k of coordinate $\mathbf{r}_j$ and $\mathbf{r}_k$, respectively. V denotes the volume of normalization and $\varphi_{Cb}(\mathbf{r})$ is the Coulomb solution [2], which is the wavefunction of the relative motion in repulsive Coulomb potential. $\nabla_{\mathbf{R}}^2$ and $\nabla_{\mathbf{r}}^2$ are Laplace operators in the CM and relative coordinates, $\mathbf{K}$ is the wave vector of the CM motion and $E_{jk} = E_{CM} + E$ with $E_{CM} = \hbar^2\mathbf{K}^2/[2m_0(A_j + A_k)]$ and $E = \hbar^2\mathbf{k}^2/(2m_0 a_{jk})$. e is the elementary charge with $e^2 = \alpha_f \hbar c$.

The contact probability density in the nuclear volume is $|\varphi_{Cb}(\mathbf{0})|^2 = f_{jk}^2/V$, where

$$f_{jk} = \left| e^{-\pi\eta_{jk}/2}\Gamma(1 + i\eta_{jk}) \right| = \sqrt{\frac{2\pi\eta_{jk}}{\exp(2\pi\eta_{jk}) - 1}}. \tag{7}$$

The cross section (1) represents the result of a first order calculation of standard perturbation theory of quantum mechanics, that is proportional to f_{jk}^2. The exponential factor in (1) comes from f_{jk}^2 in the $E \rightarrow 0$ limit. Accordingly, the magnitude of the factor f_{jk} is crucial from the point of view of magnitude of the cross section.

In the case of an exothermal nuclear reaction, i.e. if the reaction energy $\Delta > 0$ (Δ equals the difference between initial and final rest energies), the spontaneous process is allowed by energy conservation but with $E \rightarrow 0$ the process becomes forbidden due to Coulomb repulsion which is mathematically manifested in $\lim_{E\rightarrow 0} \sigma(E) = 0$ due to $\lim_{E\rightarrow 0} f_{jk}^2(E) = 0$ resulting $\lim_{E\rightarrow 0} |\varphi_{Cb}(\mathbf{0})|^2 = 0$ too. Consequently one can say that because of $\lim_{E\rightarrow 0} |\varphi_{Cb}(\mathbf{0})|^2 = 0$ the expected reaction rate becomes zero in the $E \rightarrow 0$ limit. (If one of the reacting particle is neutral, which is the case of neutron capture processes, the cross section has non zero value in the $E \to 0$ limit, see e.g. thermal neutron absorption cross sections [3].)

Experience in atomic physics indicates that in case of forbidden transitions the second order process may play an important role. As e.g. in the case of the hydrogen $2s_{1/2} - 1s_{1/2}$ transition, which is a forbidden electric dipole transition, the largest transition rate comes from a two photonic process [4] in which the sum of the energies of the simultaneously emitted photons equals the difference between the energies of states $2s_{1/2}$ and $1s_{1/2}$. The mean life time $1/7$ s of the $2s_{1/2}$ state due to the two photonic process is much longer than the lifetime 1.6×10^{-9} s of state $2p_{1/2}$ for which electric dipole transition is allowed. Thus one can conclude that a second order process from the point of view of perturbation calculation can result small but finite transition rate. In the second order process the state is changed in first order and states, which can produce allowed electric dipole transition rate, are mixed with small amplitude to the initial $2s_{1/2}$ state meanwhile two particles are emitted.

II. STATEMENT AND EXAMPLES

Similarly to what was said above an essential change of the initial eigenstate of (4) of $E = 0$ may happen due to *any perturbation* since it can mix states of $E \neq 0$ with small but finite amplitude to the initial state resulting much smaller (compared to neutron absorption) but finite rate of the nuclear reaction originally forbidden in the $E \to 0$ limit. Consequently, cross section and rate of processes to be considered should be calculated by the rules of standard perturbation calculation of quantum mechanics. Our statement applies to every nuclear process for which $\lim_{E\to 0} \sigma(E) = 0$ holds, and as such it concerns low energy nuclear physics with charged participants in general.

Since the above statement is quite general it is only illustrated by modification of forbidden nuclear reactions due to impurities in plasma. We demonstrate the mechanism on the

$$ {}^{A_1}_{z_1}V + {}^{A_2}_{z_2}w + {}^{A_3}_{z_3}X \to {}^{A_1}_{z_1}V' + {}^{A_3+A_2}_{z_3+z_2}Y + \Delta \qquad (8) $$

and

$$ {}^{A_1}_{z_1}V + {}^{A_2}_{z_2}w + {}^{A_3}_{z_3}X \to {}^{A_1}_{z_1}V' + {}^{A_4}_{z_4}Y + {}^{A_5}_{z_5}W + \Delta \quad (9) $$

processes. Reaction (8) is an impurity $\left({}^{A_1}_{z_1}V\right)$ assisted capture of particle ${}^{A_2}_{z_2}w$, e.g. capture of proton (p), deuteron (d), triton (t), ${}^{3}He$, ${}^{4}He$, etc. The reaction (9) with two final fragments is possible with conditions $A_2 + A_3 = A_4 + A_5$ and $z_2 + z_3 = z_4 + z_5$. The reaction energy Δ is the difference between the sum of the initial and final mass excesses, i.e. $\Delta = \Delta_{A_2,z_2} + \Delta_{A_3,z_3} - \Delta_{A_3+A_2,z_3+z_2}$ in case of (8) and $\Delta = \Delta_{A_2,z_2} + \Delta_{A_3,z_3} - \Delta_{A_4,z_4} - \Delta_{A_5,z_5}$ in case of (9) where Δ_{A_j,z_j} and $\Delta_{A_3+A_2,z_3+z_2}$ are the corresponding mass excesses [5]. Since particle 1 merely assists the nuclear reaction its rest mass does not change.

Usually capture of particle ${}^{A_2}_{z_2}w$ may happen in the ${}^{A_2}_{z_2}w + {}^{A_3}_{z_3}X \to {}^{A_3+A_2}_{z_3+z_2}Y + \gamma$ (with $\Delta > 0$) reaction where γ emission is required by energy and momentum conservation. Accordingly (8) describes a new type of ${}^{A_2}_{z_2}w$-capture. In the usual ${}^{A_2}_{z_2}w$-capture reaction particles ${}^{A_3+A_2}_{z_3+z_2}Y$ and γ take away the reaction energy and the reaction is governed by electromagnetic interaction. In reaction (8) the reaction energy is taken away by particles ${}^{A_1}_{z_1}V'$ and ${}^{A_3+A_2}_{z_3+z_2}Y$ while the reaction is governed by Coulomb as well as strong interactions.

Since in (8) and (9) the reaction energy is taken away by particles ${}^{A_1}_{z_1}V'$, ${}^{A_3+A_2}_{z_3+z_2}Y$ and ${}^{A_1}_{z_1}V'$, ${}^{A_4}_{z_4}Y$, ${}^{A_5}_{z_5}W$, respectively, as their kinetic energy that they can lose in a very short range to their environment converting the reaction energy efficiently into heat, their direct observation is difficult.

III. MECHANISM AND MODEL

It is assumed that in a plasma all components are in atomic, atom-ionic or fully-ionized state while the necessary number of electrons required for electric neutrality are also present. Three screened charged heavy particles of rest masses m_j and nuclear charges $z_j e$ ($j = 1, 2, 3$) are taken. The total Hamiltonian which describes this 3-body system is

$$ H_{tot} = H_{kin,1} + H_{23,sc} + V_{Cb,sc}(1,2) + V_{Cb,sc}(1,3), \quad (10) $$

where $H_{23,sc} = H_{kin,2} + H_{kin,3} + V_{Cb,sc}(2,3)$ is the Hamiltonian of particles 2 and 3 whose nuclear reaction will be discussed. $H_{kin,j}$ denotes the kinetic Hamiltonian of particle j and particle 1 is considered to be free.

$$ V_{Cb,sc}(j,k) = \frac{z_j z_k e^2}{2\pi^2} \int \frac{\exp(i\mathbf{q}\mathbf{r}_{jk})}{q^2 + q^2_{sc,jk}} d\mathbf{q}, \qquad (11) $$

denotes the screened Coulomb interaction between particles j and k with screening parameter $q_{sc,jk}$.

It is supposed that stationary solutions $|1\rangle$ and $|2,3\rangle_{sc}$ of energy eigenvalues E_1 and E_{23} of the stationary Schrödinger equations $H_{kin,1}|1\rangle = E_1|1\rangle$ with E_1 the kinetic energy of particle 1 and $H_{23,sc}|2,3\rangle_{sc} = E_{23}|2,3\rangle_{sc}$ with $E_{23} = E_{CM} + E$ are known. Here E and E_{CM} are the energies attached to the relative and CM motions (of wave numbers $\mathbf{k}$ and $\mathbf{K}$) of particles 2 and 3. Thus H_{tot} can be written as $H_{tot} = H_0 + H_{Int}$ with $H_0 = H_1 + H_{23,sc}$ as the unperturbed Hamiltonian and

$$ H_{Int} = V_{Cb,sc}(1,2) + V_{Cb,sc}(1,3) \qquad (12) $$

as the interaction Hamiltonian (time independent perturbation). The stationary solution $|1,2,3\rangle_{0,sc}$ of $H_0|1,2,3\rangle_{0,sc} = E_0|1,2,3\rangle_{0,sc}$ with $E_0 = E_1 + E_{23}$ can be written as $|1,2,3\rangle_{0,sc} = |1\rangle|2,3\rangle_{sc}$ which is the direct product of states $|1\rangle$ and $|2,3\rangle_{sc}$. The states $|1,2,3\rangle_{0,sc}$ form complete system. The approximate solution of

$H_{tot}\left|1,2,3\right\rangle_{sc} = E_0\left|1,2,3\right\rangle_{sc}$ in the screened case is obtained with the aid of standard time independent perturbation calculation [6] and the first order approximation is expanded in terms, which are called intermediate states, of the complete system $\left|1,2,3\right\rangle_{0,sc}$.

The solutions $\left|2,3\right\rangle_{sc}$ in the screened case are unknown (their coordinate representation $\left\langle \mathbf{R},\mathbf{r}\left|2,3\right\rangle_{sc}\right.$ is denoted by $\varphi_{23}\left(\mathbf{R},\mathbf{r}\right)_{sc}$) but the solution of $H_{23}\left|2,3\right\rangle = E_{23}\left|2,3\right\rangle$ in the unscreened case is known and the coordinate representation $\left\langle \mathbf{R},\mathbf{r}\left|2,3\right\rangle\right. = \varphi_{23}\left(\mathbf{R},\mathbf{r}\right)$ of $\left|2,3\right\rangle$, as it is said above, has the form $\varphi_{23}\left(\mathbf{R},\mathbf{r}\right) = V^{-1/2}e^{i\mathbf{KR}}\varphi_{Cb}(\mathbf{r})$, where $\varphi_{Cb}(\mathbf{r})$ is the unscreened Coulomb solution [2] (now $\mathbf{r} = \mathbf{r}_{23}$).

The two important limits of $\varphi_{23}\left(\mathbf{R},\mathbf{r}\right)_{sc}$ are: the solution $\varphi_{23}\left(\mathbf{R},\mathbf{r},nucl\right)_{sc}$ in the nuclear volume and the solution $\varphi_{23}\left(\mathbf{R},\mathbf{r},out\right)_{sc}$ in the screened regime. In the nuclear volume screening is negligible thus $\varphi_{23}\left(\mathbf{R},\mathbf{r}\right)_{sc} = \varphi_{23}\left(\mathbf{R},\mathbf{r}\right)$. Furthermore, in this case in $\varphi_{23}\left(\mathbf{R},\mathbf{r}\right)$ an approximate form $\varphi_{Cb,a}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}f_{23}(|\mathbf{k}|)/\sqrt{V}$ of the (unscreened) Coulomb solution $\varphi_{Cb}(\mathbf{r})$ may be used. Here $f_{23}(|\mathbf{k}|)$ is the appropriate factor given by (7) corresponding to particles 2 and 3. Thus $\varphi_{23}\left(\mathbf{R},\mathbf{r},nucl\right)_{sc} = f_{23}(|\mathbf{k}|)e^{i\mathbf{KR}}e^{i\mathbf{kr}}/V$ is used in the range of the nucleus and in the calculation of the nuclear matrix-element. In the screened (outer) range, where Coulomb potential is negligible, the solution becomes $\varphi_{23}\left(\mathbf{R},\mathbf{r},out\right)_{sc} = e^{i\mathbf{KR}}e^{i\mathbf{kr}}/V$ that is used in the calculation of the Coulomb matrix element.

In the screened range the initial wave function of zero energy is $\varphi_i = V^{-3/2}$. The intermediate states of particles 2 and 3 are determined by the wave number vectors $\mathbf{K}$ and $\mathbf{k}$. In the case of the assisting particle 1 the intermediate and final state is a plane wave of wave number vector $\mathbf{k}_1$.

The matrix elements $V_{Cb,\nu i}$ of the screened Coulomb potential between the initial and intermediate states are

$$V_{Cb}(1,s)_{\nu i} = \frac{z_1 z_s}{2\pi^2}e^2\frac{(2\pi)^9}{V^3}\delta\left(\mathbf{k}_1+\mathbf{K}\right)\times \\ \times\frac{\delta\left(\mathbf{k}+a(s)\mathbf{k}_1\right)}{\mathbf{k}_1^2+q_{sc,1s}^2} \quad (13)$$

where $a(s) = \left(-A_3\delta_{s,2}+A_2\delta_{s,3}\right)/\left(A_2+A_3\right)$ and $s = 2,3$.

IV. CHANGE OF THREE-PARTICLE WAVEFUNCTION IN NUCLEAR RANGE

According to standard time independent perturbation theory of quantum mechanics [6] the first order change of the wavefunction in the range $r \lesssim R_0$ (R_0 is the nuclear radius of particle 3) due to screened Coulomb perturbation is determined as

$$\delta\varphi\left(\mathbf{r}\right) = \sum_{s=2,3}\delta\varphi\left(s,\mathbf{r}\right) \quad (14)$$

with

$$\delta\varphi\left(s,\mathbf{r}\right) = \int\int\frac{V_{Cb}(1,s)_{\nu i}}{E_\nu - E_i}\frac{V}{(2\pi)^6}\times \\ \times e^{i(\mathbf{KR}+\mathbf{k_1 r_1})}\varphi_{Cb,a}(\mathbf{k},\mathbf{r})d\mathbf{K}d\mathbf{k}, \quad (15)$$

where E_i and E_ν are the kinetic energies in the initial and intermediate states, respectively. The initial momenta and kinetic energies of particles 1, 2 and 3 are zero ($E_i = 0$) and $E_\nu = E_{23} + \hbar^2\mathbf{k}_1^2/\left(2m_0A_1\right)$. Thus

$$\delta\varphi\left(s,\mathbf{r}\right) = z_1 z_s\alpha_f\frac{4\pi\hbar c}{V^{5/2}}\frac{e^{i(\mathbf{k_1 r_1}-\mathbf{k}_1\mathbf{R})}}{\mathbf{k}_1^2+q_{sc,1s}^2}\times \\ \times\frac{2m_0a_{1s}}{\hbar^2\mathbf{k}_1^2}\left[f_{23}\left(k\right)e^{i\mathbf{kr}}\right]_{\mathbf{k}=a(s)\mathbf{k}_1}. \quad (16)$$

It can be seen that the arguments of $f_{23}\left(|\mathbf{k}|\right)$ are $|\mathbf{k}| = \frac{A_3}{A_2+A_3}k_1$ and $|\mathbf{k}| = \frac{A_2}{A_2+A_3}k_1$, here $k_1 = |\mathbf{k}_1|$. Consequently, if particle 1 obtains large kinetic energy, as is the case in nuclear reactions (e.g. $\mathbf{k}_1^2 = k_0^2 = 2m_0a_{14}\Delta\hbar^{-2}$ in the case of reaction (8)), then the factors $f_{23}\left(|\mathbf{k}|\right)$ and the rate of the process too will be considerable. (In this case one can neglect $q_{sc,jk}^2$ in the denominator of (16)). Since $\lim_{E\to 0}\left|\delta\varphi\left(\mathbf{0}\right)\right|^2 \neq 0$, i.e. it remains finite in the $E \to 0$ limit, and the expected reaction rate too remains finite. Furthermore, $\delta\varphi\left(\mathbf{r}\right)$, which causes the effect, is temperature independent. Up to this point the calculation and the results are nuclear reaction and nuclear model independent.

V. CROSS SECTION

When calculating the cross section of reaction ${}_{z_1}^{A_1}V + p + d \rightarrow {}_{z_1}^{A_1}V' + {}_2^3He + 5.493MeV$ the Hamiltonian $V_{st}\left(2,3\right) = -V_0$ if $|\mathbf{r}_{23}| = |\mathbf{r}| \leq b$ and $V_{st}\left(2,3\right) = 0$ if $|\mathbf{r}_{23}| = |\mathbf{r}| > b$ of strong interaction which is responsible for nuclear reaction between particles 2 and 3 is used. For the final state of the captured proton the Weisskopf-approximation is applied, i.e. $\Phi_f\left(\mathbf{r}\right) = \Phi_{fW}\left(\mathbf{r}\right)$ with $\Phi_{fW}\left(\mathbf{r}\right) = \sqrt{3/\left(4\pi R_0^3\right)}$ if $r \leq R_0$, and $\Phi_{fW}\left(\mathbf{r}\right) = 0$ for $r > R_0$, where R_0 is the nuclear radius. We take $V_0 = 25$ MeV and $R_0 = b = 2\times 10^{-13}$ cm [3] in the case of pd reaction.

The matrix element of the potential of the strong interaction between intermediate $\left(e^{i\mathbf{KR}}\varphi_{Cb,a}(\mathbf{k},\mathbf{r})/\sqrt{V}\right)$ and final $\left(e^{i\mathbf{k}_4\cdot\mathbf{R}}\Phi_f\left(\mathbf{r}\right)/\sqrt{V}\right)$ states and in the Weisskopf-approximation is

$$V_{st,f\nu}^W = -V_0\frac{\sqrt{12\pi R_0}}{k}f_{23}(k)H\left(k\right)\frac{(2\pi)^3}{V^{3/2}}\delta\left(\mathbf{K}-\mathbf{k}_4\right) \quad (17)$$

where $H\left(k\right) = \int_0^1\sin(kR_0x)xdx$. According to standard time independent perturbation theory of quantum mechanics [6] the transition probability per unit time

$\left(W_{fi}^{(2)}\right)$ of the process can be written as

$$W_{fi}^{(2)} = \frac{2\pi}{\hbar} \int\int \left| T_{fi}^{(2)} \right|^2 \delta(E_f - \Delta) \frac{V^2}{(2\pi)^6} d\mathbf{k}_1 d\mathbf{k}_4 \quad (18)$$

with

$$T_{fi}^{(2)} = \int\int \sum_{s=2,3} \frac{V_{st,f\nu} V_{Cb}(1,s)_{\nu i}}{E_\nu - E_i} \frac{V^2}{(2\pi)^6} d\mathbf{K} d\mathbf{k}. \quad (19)$$

Substituting everything obtained above into (19) and (18), where E_f is the sum of kinetic energies of the final particles (1 and 4), one can calculate $W_{fi}^{(2)}$. The cross section $\sigma_{23}^{(2)}$ of the process is defined as $\sigma_{23}^{(2)} = N_1 W_{fi}^{(2)} / (v_{23}/V)$ where N_1 is the number of particles 1 in the normalization volume V and v_{23}/V is the flux of particle 2 of relative velocity v_{23}.

$$v_{23}\sigma_{23}^{(2)} = n_1 S_{pd} \quad (20)$$

where $n_1 = N_1/V$ is the number density of particles 1 and

$$S_{pd} = 24\pi^2 \sqrt{2} c R_0 \frac{z_1^2 \alpha_f^2 V_0^2 (\hbar c)^4}{\Delta^{9/2} (m_0 c^2)^{3/2}} \times \times \frac{(A_2 + A_3)^2}{a_{14}^{7/2}} \left[F(2) + F(3) \right]^2 \quad (21)$$

with

$$F(s) = \frac{z_s a_{1s}}{A_3 \delta_{s,2} + A_2 \delta_{s,3}} f_{23} \left[a(s) k_0 \right] H \left[a(s) k_0 \right], \quad (22)$$

$s = 2,3$ and $k_0 = \hbar^{-1} \sqrt{2 m_0 a_{14} \Delta}$.

The cross section $\sigma_{23}^{(2)}$ of the process ${}_{z_1}^{A_1}V + p + d \rightarrow {}_{z_1}^{A_1}V' + {}_2^3He + 5.493 MeV$ is $\sigma_{23}^{(2)} = n_1 S_{pd}/v_{23}$, where $S_{pd} = 1.89 \times 10^{-53} z_1^2 \ cm^6 s^{-1}$ with z_1 the charge number of the assisting nucleus. $\sigma_{23}^{(2)}$, similarly to thermal neutron capture cross sections, has $1/v_{23}$ dependence. In case of 0.025 eV initial kinetic energy and with $z_1 = 54$ (Xe) $\sigma_{23}^{(2)} = n_1 \times 2.5 \times 10^{-31}$ b from which $\sigma_{23}^{(2)} = 0.0066$ nb at $n_1 = 2.65 \times 10^{19}$ cm^{-3} which is the number density of an atomic gas of normal state.

In the case of reactions with two final fragments (see (9)) the nuclear matrix element can be derived from $S(E)$ (see (1)), i.e. in long wavelength approximation from $S(0)$ which is the astrophysical S-factor at $E = 0$, in the following manner.

Calculating the transition probability per unit time $W_{fi}^{(1)}$ of the usual (first order) process in standard manner

$$W_{fi}^{(1)} = \int \frac{2\pi}{\hbar} \left| V_{st,fi} \right|^2 \delta (E_f - \Delta) \frac{V}{(2\pi)^3} d\mathbf{k}_f, \quad (23)$$

where $\mathbf{k}_f$ is the relative wave number of the two fragments of rest masses $m_4 = m_0 A_4$, $m_5 = m_0 A_5$ and atomic numbers A_4, A_5, and $E_f = \hbar^2 \mathbf{k}_f^2 / (2 m_0 a_{45})$ is the sum of their kinetic energy. For the magnitude of nuclear matrix element $V_{st,fi}$ we take the form $|V_{st,fi}| = f_{23}(k_i) |h_{fi}| / V$, where $f_{23}(k_i)$ is the Coulomb factor of the initial particles 2 and 3 with k_i the magnitude of their relative wave number vector $\mathbf{k}_i$. (The Coulomb factor $f_{45}(k_f) \approx 1$ of the final particles 4 and 5 with k_f the magnitude of their relative wave number vector $\mathbf{k}_f$.) It is supposed that $|h_{fi}|$ does not depend on $\mathbf{k}_i$ and $\mathbf{k}_f$ namely the long wavelength approximation is used. In this case the product of the relative velocity v_{23} of the initial particles 2, 3 and the cross section $\sigma_{23}^{(1)}$ is

$$v_{23}\sigma_{23}^{(1)} = \frac{|h_{fi}|^2 f_{23}^2 (k_i) (m_0 a_{45})^{3/2} \sqrt{2\Delta}}{\pi \hbar^4}. \quad (24)$$

On the other hand, $v_{23}\sigma_{23}^{(1)}$ is expressed with the aid of (1) and $v_{23} = \sqrt{2E/(m_0 a_{23})}$. From the equality of the two kinds of $v_{23}\sigma_{23}^{(1)}$ one gets

$$|h_{fi}|^2 = \frac{(\hbar c)^4 S(0)}{z_2 z_3 \alpha_f (m_0 c^2)^{5/2} \sqrt{2\Delta} a_{45}^{3/2} a_{23}}. \quad (25)$$

In the case of the impurity assisted, second order process $|V_{st,f\nu}| = f_{23}(k) |h_{fi}| (2\pi)^3 \delta (\mathbf{K} - \mathbf{K}_f) / V^2$ where $\mathbf{K}_f$ and $\mathbf{k}_f$ are the final wave number vectors attached to CM and relative motions of the two final fragments, particles 4 and 5. $\mathbf{k}_f$ appears in E_f in the energy Dirac-delta. Repeating the calculation of the transition probability per unit time of the impurity assisted, second order process applying the above expression of $|V_{st,f\nu}|$ one gets

$$v_{23}\sigma_{23}^{(2)} = n_1 S_{'reaction'}, \quad (26)$$

where $\sigma_{23}^{(2)}$ is the cross section of the process and

$$S_{'reaction'} = \frac{8 \alpha_f^2 z_1^2}{a_{23} a_{123}^3} \frac{S(0) c}{m_0 c^2} \left(\frac{\hbar c}{\Delta} \right)^3 I \quad (27)$$

with

$$I = \int_0^1 \left(\sum_{s=2,3} \frac{z_s a_{1s} \sqrt{A_s}}{\sqrt{e^{b_{23} A_s \frac{1}{x}} - 1}} \right)^2 \frac{\sqrt{1 - x^2}}{x^7} dx. \quad (28)$$

Here $b_{23} = 2\pi z_2 z_3 \alpha_f \sqrt{m_0 c^2 / (2 a_{123} \Delta)}$ with $a_{123} = A_1 (A_2 + A_3) / (A_1 + A_2 + A_3)$. In the index $'reaction'$ the reaction resulting the two fragments will be marked (see Table I.).

It is plausible to extend the investigation to the possible consequence of plasma-wall interaction. The role of particle 1 is played by the wall which is supposed to be a solid (metal) from atoms with nuclei of charge and mass numbers $z_1$and A_1. For initial state a Bloch-function of the form

$$\varphi_{\mathbf{k}_{1,i}}(\mathbf{r}_1) = N_1^{-1/2} \sum_{\mathbf{L}} e^{i \mathbf{k}_{1,i} \cdot \mathbf{L}} a(\mathbf{r}_1 - \mathbf{L}) \quad (29)$$

Reaction	$S(0)$	$S_{'Reaction'}$	Δ	$p_{'Reaction'}$
$d(d,n)^3_2He$	0.055	1.01×10^{-48}	3.269	9.82
$d(d,p)t$	0.0571	1.10×10^{-48}	4.033	13.2
$d(t,n)^4_2He$	11.7	1.06×10^{-46}	17.59	5.57×10^{3}
$^3_2He(d,p)^4_2He$	5.9	1.51×10^{-48}	18.25	82.6
$^6_3Li(p,\alpha)^3_2He$	2.97	1.99×10^{-49}	4.019	2.38
$^6_3Li(d,\alpha)^4_2He$	16.9	1.33×10^{-49}	22.372	8.84
$^7_3Li(p,\alpha)^4_2He$	0.0594	3.85×10^{-51}	17.347	0.199
$^9_4Be(p,\alpha)^6_3Li$	17	1.79×10^{-49}	2.126	1.13
$^9_4Be(p,d)^8_4Be$	17	1.66×10^{-49}	0.56	0.277
$^9_4Be(\alpha,n)^{12}_6C$	2.5×10^{3}	6.22×10^{-51}	5.701	0.106
	6×10^{5}	1.49×10^{-48}	5.701	25.4
$^{10}_5B(p,\alpha)^7_4Be$	4	1.04×10^{-50}	1.145	0.0356
	2×10^{3}	5.21×10^{-48}	1.145	17.8
$^{11}_5B(p,\alpha)^8_4Be$	187	5.16×10^{-49}	8.59	13.2

TABLE I: $S_{'reaction'}$ and power density of Xe assisted reactions with two final fragments in long wavelength approximation. $S(0)$ is the astrophysical S-factor at $E=0$ in $MeVb$ [1], [8], [9]. $S_{'Reaction'}$ (in cm^6s^{-1}) is calculated using (27) with (28) taking $z_1=54$ (Xe), Δ is the energy of the reaction in MeV and $p_{'Reaction'}=n_1n_2n_3S_{'Reaction'}\Delta$ is the power density in Wcm^{-3} that is calculated with $n_1=n_2=n_3=2.65\times10^{20}$ cm^{-3}. In the case of $^9_4Be(\alpha,n)^{12}_6C$ and $^{10}_5B(p,\alpha)^7_4Be$ reactions the astrophysical S-factor [$S(E)$] has strong energy dependence therefore the calculation was carried out with two characteristic values of $S(E)$.

is taken, that is localized around all of the lattice points [7]. Here $\mathbf{r}_1$ is the coordinate, $\mathbf{k}_{1,i}$ is wave number vector of the first Brillouin zone (BZ) of the reciprocal lattice, $a(\mathbf{r}_1-\mathbf{L})$ is the Wannier-function, which is independent of $\mathbf{k}_{1,i}$ within the BZ and is well localized around lattice site $\mathbf{L}$. N_1 is the number of lattice points of the lattice of particles 1. Repeating the cross section calculation applying Bloch-function it is obtained that cross section results remain unchanged and $n_1=N_{1c}/v_c$, where v_c is the volume of elementary cell of the solid and N_{1c} is the number of particles 1 in the elementary cell.

VI. RATE AND POWER DENSITIES

The rate in volume V is

$$\frac{dN_{'reaction'}}{dt}=N_3\Phi_{23}\sigma^{(2)}_{23}, \tag{30}$$

where $\Phi_{23}=n_2v_{23}$ is the flux of particles 2 with $n_2=N_2/V$ their number density. N_2 and N_3 are the numbers of particles 2 and 3 in the normalization volume. The rate and power densities are defined as

$$r_{'reaction'}=\frac{1}{V}\frac{dN_{'reaction'}}{dt}=n_3n_2n_1S_{'reaction'} \tag{31}$$

and

$$p_{'reaction'}=r_{'reaction'}\Delta=n_1n_2n_3S_{'reaction'}\Delta, \tag{32}$$

respectively, where $n_3=N_3/V$ is the number density of particles 3. $r_{'reaction'}$ and $p_{'reaction'}$ are both temperature independent.

The rate (r_{pd}) and power (p_{pd}) densities of reaction $^{A_1}_{z_1}V+p+d\rightarrow{}^{A_1}_{z_1}V'+{}^3_2He$ are determined taking $z_1=54$ (Xe) and $n_1=n_2=n_3=2.65\times10^{20}$ cm^{-3} (n_1, n_2 and n_3 are the number densities of Xe, p and d, i.e. particles 1, 2 and 3) for which considerable values are obtained: $r_{pd}=1.02\times10^{12}$ $cm^{-3}s^{-1}$ and $p_{pd}=0.901$ Wcm^{-3}. If the impurity is Hg or U then these numbers must be multiplied by 2.2 or 2.9, respectively.

The results of $S_{'reaction'}$ and power density calculations of a number of Xe assisted reactions with two final fragments in long wavelength approximation and with $n_1=n_2=n_3=2.65\times10^{20}$ cm^{-3} can be found in Table I.

VII. CONCLUSIONS

It is found that *any perturbation* may lead to finite cross section and rate of nuclear reactions forbidden in the $E\rightarrow0$ limit. Since this statement applies to every nuclear process forbidden in the $E\rightarrow0$ limit it concerns low energy nuclear physics with charged participants in general.

These findings have important bearing on energy production. It is commonplace that nuclear fusion reactors need to be heated to very high temperature to overcome the Coulomb repulsion between nuclei to fuse and it is also assumed that in some (e.g. tokamak-like) devices the presence of impurities during the heat up and working periods is undesirable because of high power loss generated by them [10]. However it is shown here that spectator nuclei can allow new types of reactions for which both rate and power densities are temperature independent. What is more remarkable, the mechanism found does not need plasma state to work at all which bring up the possibility of a quite new type of apparatus working at much lower temperature than the temperature of fusion power stations planned to date. On the other hand, the density of the components has to be considerably increased. The effective influence of wall-gas mix interaction brings up the possible importance of gas mix-metal surface processes too. Moreover, investigating mass excess data [5] one can recognize that in the case of (8) and (9) the number of energetically allowed reactions is very large. Based on these results it may be expected that search for new approach to energy production by nuclear fusion may be started.

Finally, it may be stated that a very great number of reactions, which are determined by different initial states, different perturbations and different processes of second and higher order and which may be attached to forbidden reactions, have not been investigated up till now.

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